NEMP PROBE STUDY

Language used by students in mathematics for quantitative and numerical comparisons

Report of a Probe Study carried out for the National Education Monitoring Project.

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Executive Summary

- The language used by Year 4 and Year 8 students from different decile groups, both genders and Pasifika and non–Pasifika low decile students was analysed when they were responding four tasks involving mathematics.
- The main features included in mathematical explanations were Premises, Consequences and Conclusions.
- Other appropriate features included Introductions, Suppositions and Elaborators.
- By and large, students who gave correct answers used more appropriate language.
- The pattern of use of these structures amongst the subgroups differed with the questions being answered. Where differences were notable, Year 8 boys from high decile schools were most proficient in the use of these mathematical linguistic structures.
- While there were few differences between Pasifika and non-Pasifika students from low decile schools, Pasifika students were less likely to use Premises or Consequences in their explanations.
- Appropriate mathematical responses were seen to depend upon appropriate mathematical language, knowledge of how to interact with a teacher and knowledge of how to structure mathematical explanations or justifications.
- It is recommended that teachers ensure that students use the linguistic elements in their mathematical explanations.

Chapter 1

Language and mathematics: The role of explanations and justifications

This report documents the language used by students in Year 4 and Year 8 in responding to mathematical assessment tasks which required them to do numerical or quantitative comparisons. It also describes some differences which were found between students of different ages, genders and the decile level of the schools they attended. This leads to a more extended discussion on story telling within mathematics and the text structures of mathematics explanations and justifications.

In learning mathematics, language is the vehicle through which mathematical ideas are described by the teacher but it is also the medium that students use to discuss these ideas and make sense of them (Kaput, 1988). In order to evaluate students' understandings, teachers often listen and read students' responses to tasks. For example, Moskal and Magone (2000) suggested that previous research had found that 'students' written explanations to well-designed tasks can provide robust accounts of their mathematical reasoning' (p. 313) and so can be used by teachers to assess students' knowledge. For students to make the most of language as a tool for their thinking and to adequately explain what they know, they need to learn how to express themselves mathematically (Chapman, 1997).

Although there has been much research on the mathematics register and teachers' use of language, especially in regard to questions (Martino & Maher, 1994), little research has been done to document what is typical of the language used by students at different stages (see for example Ellerton & Clarkson, 1996). Yet curriculum documents such as the *Mathematics in the New Zealand Curriculum* (Learning Media, 1992) abound with statements that students should 'develop the skills and confidence to use their own language, and the language of mathematics, to express mathematical ideas' (p. 23). Being able 'to make predictions, formulate generalizations, justify their thinking' (Burns, 1985, p. 17) is believed to support students' development of mathematical thinking. However, as Kristy and Chval's (2002) research clearly demonstrated, it is not always apparent to teachers when everyday language is acceptable and when students should be encouraged to use mathematical language. In investigating students' writing about mathematics, Morgan (1998) felt that there was a general lack of knowledge about language and language teaching. As a result she was unsure that students could adequately express themselves mathematically. This is supported by research by Bicknell (1999) in which New Zealand secondary teachers voiced their belief that the process of writing explanations and justifications should be explicitly taught to students. This research sets out to document the language used by Year 4 and Year 8 students in giving mathematical explanations and justifications of quantitative and numerical comparisons. By knowing what language students typically use at different ages to talk about mathematics, teachers would be able to develop their students' fluency in as well as formal knowledge of mathematical language.

Students' explanations and justifications is one area of mathematical language which has received attention as a way of improving their learning of mathematics (see Forman, Larreamendy-Joerns, Stein & Browns, 1998). In New Zealand, the advent of the National Certificate in Educational Achievement has emphasised the need for students to be able to write explanations and justifications in order to gain merits or excellences in their assessments (Irwin & Niederer, 2002; Meaney, 2002). These examples of mathematical genres are perceived as leading onto developing mathematical proofs (Bicknell, 1999). Bicknell (1999), using the work of Thomas (1973), gave these definitions:

> [a]n *explanation* can be defined as making clear or telling why a state of affairs or an occurrence exists or happens, whereas a *justification* provides grounds, evidence or reasons to convince others (or persuade ourselves) that a claim or assertion is true.

Another who has done research on students' explanations and justifications is Erna Yackel (2001), who using a symbolic interaction perspective, stated that:

> [s]tudents and the teacher give mathematical explanations to clarify aspects of their mathematical thinking that they think might not be readily apparent to others. They give mathematical justifications in response to challenges to apparent violations of normative mathematical activity.

Using elements from both perspectives, we describe a mathematical explanation as the description of *what* was undertaken to solve a problem (the solution strategy) whereas a justification would be *why* a certain strategy was adopted and its consequent result being accepted as appropriate. Although we acknowledge the importance of the social interaction, we are primarily concerned in this report with the explanations and justifications that students give in assessment situations. In these situations, there is limited interaction between the teacher and the student with the development of an explanation or justification not arising as a result of negotiation of meaning. Therefore our definitions imply that the explanations and justifications are given as a result of teacher questions in an informal or formal assessment situation.

Work by Krummheuer (1995) which described the components of students' mathematical arguments has been used by both Yackel (2001) and Forman et al. (1998). Using ideas from Toulmin (1969), Krummheuer (1995) proposed four components which were: claims; grounds; warrants; and backings. Claims are assertions of a point of view; in most cases these are the proposed solution to a task. Grounds are the unchallengeable facts from which the assertion is drawn. Warrants are the information which joins the grounds to the claims, while backings provide the context which make the warrants appropriate. Krummheuer (1995), Forman et al. (1998) and Yackel (2001) all used ideas about mathematical arguments in contexts of groups of students developing them in conjunction with their teachers. There appeared to be a need to investigate whether similar components could be identified in student responses where there was limited negotiation of meaning.

For students to successfully provide a mathematical explanation or justification as part of an assessment situation, they need an understanding of what is required mathematically and linguistically. All too often, students' linguistic abilities in mathematics are ignored. For example, Moskal and Magone (2000) developed a framework for ‘the identification of response patterns across classrooms’ (p. 315) but they did so without reference to the language needed by students. There seems to be a sense that if students know the mathematics they will be able to respond appropriately to an assessment task. Bills and Grey’s (2001) and Bills’ (2002) research on the responses given by seven to nine year olds to a request about how they did mental calculations showed a relationship between the correctness of responses and some linguistic features. These features included the use of personal pronouns (‘you’ and ‘I’), present tense and logical connectives such as ‘because’, ‘so’ and ‘if’ (Bills, 2002). Esty (1992) also stressed the importance of ‘five key logical connectives: “and”, “or”, “not”, “if … then” and “if and only if”’ which provided mathematics students with an understanding of when equations were true and therefore the limits on them as generalisations. Bills’ (2002) felt that ‘you’ was a reflection of how the students’ teachers discussed mathematics. As a result, he felt that ‘I’ rather than ‘you’ used in a general or generic response showed that students had more ownership of a generalisable concept.

Bills and Grey (2001) categorised responses into three kinds: general, generic and particular. ‘General’ responses were those which invoked a rule with little mention of actual numbers. ‘Particular’ responses were those which revolved around the specific numbers in the calculation. ‘Generic’ responses were those between the other two categories in that they made use of specific numbers but as examples of a specific rule. Students who were incorrect in their calculation were most likely to give a ‘particular’ response whilst students who were correct were most likely to give ‘non-particular’ responses (p. 153). The conclusion from both papers was that looking at the language that students used could be informative about students’ thinking. Their belief was that, as students were able to use these linguistic features in other types of explanations, the way that they expressed mathematical ideas reflected their conceptualised. For example, 80% of students who made a general response but used ‘I’ as the doer of the actions gave a correct response, suggesting that they had personalised their belief about how to do the calculation. However, this is based on the assumption that all students have the same linguistic resources to draw upon and that all students understand the value of being able to generalise in mathematics.

Figure 1 is presented to problematise the relationship between mathematics and language ability. It uses two continuums to illustrate how students’ language and mathematical knowledge interact as they respond. It is significantly harder to understand what students know in a situation where they give a minimal response or appear to answer a different question. In this situation, is it the mathematics or is it the language which is problematic for them? Some students have the mathematics skills but the language of the task can cause them confusion. Clarke (1993) proposed that placing mathematical tasks in contexts increases the linguistic demands on students without requiring more of them mathematically. Teachers can interpret these responses as resulting from poor mathematical knowledge whereas it is, in fact, as a result of poor linguistic knowledge.

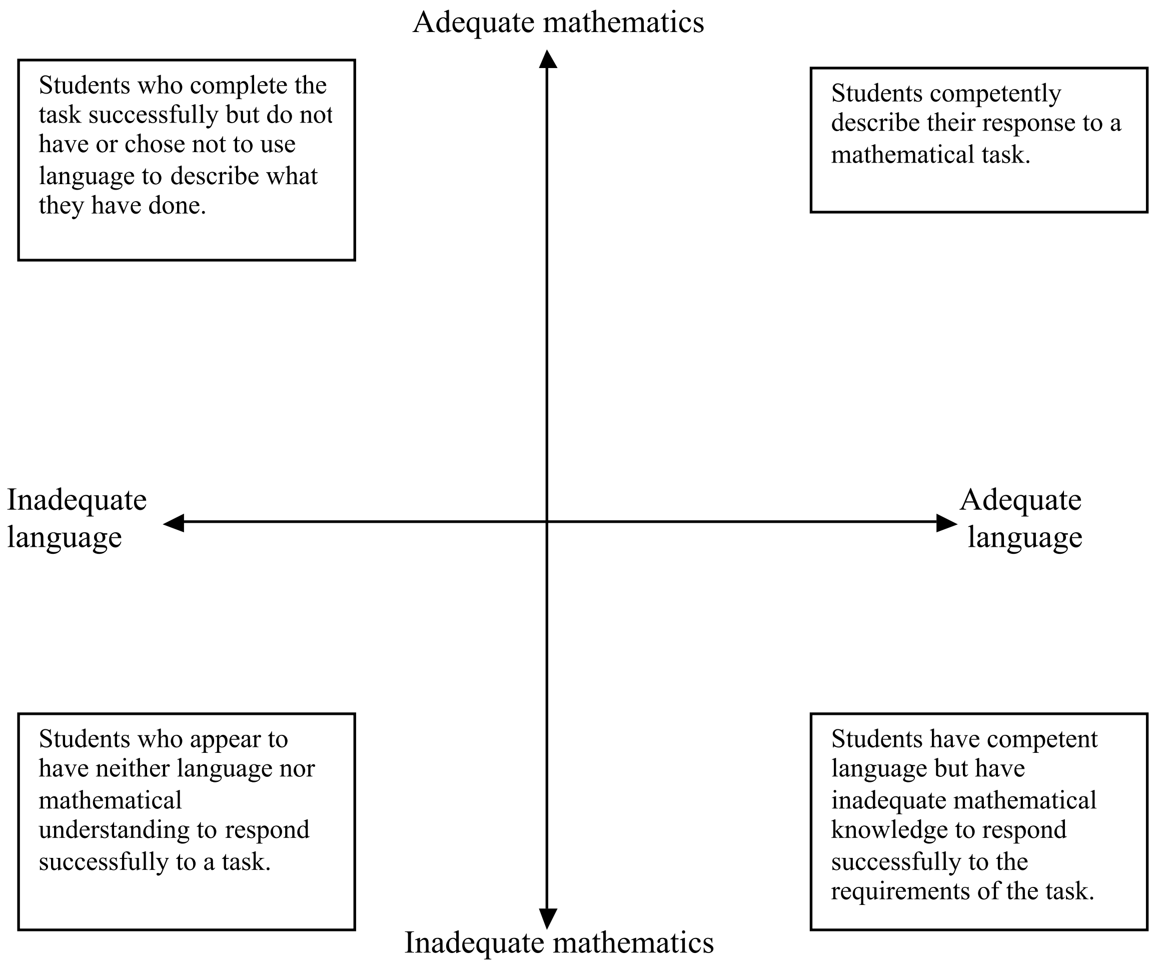


Figure 1.1: The interaction of students' mathematical and linguistic performance.

In considering to which of the quadrants in Figure 1 a particular student's response belongs, there is a need to remember that competence and performance are not equivalent. In the 1960s, Chomsky (1965) made a distinction in regard to language by stating that linguistic theory should be primarily concerned with the 'ideal speaker-listener' (p. 3). This would be similar to stating that mathematics learning theory should only concern itself with how an ideal student learns mathematics. Hymes (1972), in dismissing Chomsky's decision to ignore the outside factors which affect language performance, wrote:

> The limitations of this perspective appear when the image of the unfolding, mastering, fluent child is set beside the real children in our schools. The theory must seem, if not irrelevant, then at best a doctrine of poignancy: Poignant because of the difference between what one imagines and what one sees; poignant too, because the theory, so powerful in its own realm, cannot on its terms cope with difference. To cope with the realities of children as communicating beings requires a theory within which socio-cultural factors have an explicit and constitutive role (p. 270)

Students' lack of performance may not be due to a lack of competence but rather due to a lack of knowledge of the type of response expected in a classroom situation. This was certainly the case for the Trackton students that were the subject of Shirley Brice Heath's (1982) research in the early 1970s. Recognition of socio-cultural factors which affect the perceptions of the context is equally important when considering how students exhibit their learning in assessment activities.

Students could fail to show their mathematical knowledge because they were tired or had no personal need to answer the question (Malcolm, 1982). Aboriginal children, for example, make the decision about when they will show competency in a skill. Adults in their society would not expect them to perform to someone else's time frame, nor would adults explicitly instruct them (Christie, 1985). Instead children are expected to watch and learn and decide when they are ready to show what they can do (Harris, 1980). There is a need to discover the cultural norms in which the student is operating. Even when students cooperate with the school requirements to perform, they can unintentionally mislead their teachers about their mathematical understanding because they have reacted to other aspects of the situation.

There has been significant research to show that children with different backgrounds do not make the same choices when using language. For example, Irwin and Ginsburg (2001) showed the differences in the choice that young children from different economic backgrounds made in whether or not to use language when engaging in tasks of a mathematical nature. This would have a marked effect on their teachers' understanding of their ability. Zevenbergen (2001) described the use in two different schools of the *triadic dialogue* sequence in which the teacher asks a question, the student or students respond and the teacher then provides feedback. Bernstein's (1990) work over many years consistently supported the idea that social class would have an effect on the language choices made by students because of their different perceptions of the context and of what were appropriate choices within that context. By drawing on this work and that by others to suggest that students from lower class backgrounds were less likely to be familiar with such a classroom interaction pattern, Zevenbergen hypothesised that these students would be at a disadvantage in learning mathematics. Research on teacher perceptions of students based on their speech showed that students who spoke non-standard versions of the language of instruction were more likely to be considered to have lower ability and behave inappropriately (Haig and Oliver, 2003). It is well known that 'the understanding *that* students are expected to explain their solution is a social norm, whereas the understanding of *what counts* as an acceptable mathematical explanation is a sociomathematical norm' (Yackel, 2001, p. 14). Yet this awareness of the role of the social background in the development of mathematical explanations has not resulted in research into the mathematical explanations provided by students from different socio-cultural groups. It, therefore, seemed valuable to investigate how students from different socio-economic backgrounds used language when giving mathematical explanations and justifications.

There are also studies to show that there are differences in the language choices of boys and girls. Coates (2004), in summarising studies in this area, showed that working class girls were more likely to use standard forms as they grew older whereas boys were more likely to use a stable or increasing amount of non-standard forms. Other research has also shown that teachers nominated and interacted with boys more frequently (Wickett, 1997). It has been suggested that these interactions would have an effect on the scaffolding and modelling of language that boys and girls receive which might promote boys' ability to gain the academic language (Swann & Graddol, 1994). These studies suggest that it is worth investigating how boys and girls give mathematical explanations and justifications.

We were also aware that the acquisition of the mathematics register (or language to discuss mathematical ideas) occurs over time. Rowland (2000) found that the answers given by different-aged pupils to mathematics tasks showed differences in how uncertainty was expressed. For this research, it seemed important not only to investigate the effects of socio-economic background and gender on students' responses to tasks requiring mathematical explanations and justification, but also to look at the effect of age.

As well as looking for differences within variables such as whether younger children used different expressions to those of older children, it was also expected that combinations of variables would affect the choice of expressions. This is because no student is male or female without also being young or old and coming from a specific socio-economic background. For example, research has found that lower middle class women were more likely to imitate middle class language (Labov, 1990) and that young men were subject to more peer pressure to use a less standard form of language (Milroy, 1980 and Milroy, 1981 cited in Wodak & Benke, 1997).

It was quite clear that we were not so much interested in mathematical correctness of the explanations and justifications but how they were expressed (see Pimm & Wagner, 2003 for a discussion of the form of mathematical responses). There was a need to document how students typically structured explanations and justifications and to discover if different groups used different structures. It was then worth finding whether there was a relationship between these structures and their mathematical correctness. This was because it seemed that form could have a significant impact on knowledge being considered correct and that there was a need to have a more complete description of how groups expressed themselves. Our research question thus evolved into: What are the typical linguistic choices of different groups of students when giving mathematical explanations and justifications? In particular, we were interested in seeing if there were patterns of usage which were related to gender, socio-economic background, age and a combination of these variables.

Chapter 2

Methodology

As there was no previous research which documented students' mathematical language systematically, a methodology needed to be developed for collecting and analysing the data. In setting up this probe study, one of our original research questions had been: What is an effective method for analysing children's talk about mathematics? This chapter, therefore, sets out our decision making process in regard to choosing a methodology. Some of the decisions which would affect what we could say about students' language choices were: how much data was to be collected; from what students; interacting in which situations. As well, how we chose to interrogate the data would have an impact on the type of model of student language that could be described.

Reviewing the literature suggested that we were likely to find differences between groups of students. Variables such as gender, age and socioeconomic background can be considered as socially constructed, with language use being one of the ways that individuals are positioned within a society (Wodak & Benke, 1997). In doing this research, it was important to recognise that specific features could not be considered as 'male' or 'female' but rather if there were differences these would occur along a continuum, as 'linguistic differences are very often a matter of probabilities and tendencies' (Laver & Trugill, 1979, p. 23). As we were uncertain how differences in linguistic choices would manifest themselves, the data had to be analysed flexibly enough so that interesting things could be identified. As a quantitative approach to research requires the researcher to know what they are investigating before they begin, it was felt that a qualitative approach would be more appropriate. Qualitative research has been described as having the following 5 characteristics (Bogdan & Biklen, 1982, p. 27-30):

1. Qualitative research has the natural setting as the direct source of data and the researcher is the key instrument
2. Qualitative research is descriptive
3. Qualitative researchers are concerned with process rather than simply with outcomes or products
4. Qualitative researchers tend to analyze their data inductively
5. "Meaning" is of essential concern to the qualitative approach

However, as we wanted to produce a description of students' language, we anticipated that there would be particular features that required counting and so we did not discount the need to use some statistical techniques in our analysis. This combination of techniques from both approaches is not uncommon in research on language in educational settings (Swann, 1994). However, any combination of techniques results in compromises and some of the decisions made about the research and the related compromises are outlined below.

Data collection

In data collection, there were several issues which needed to be considered. These included: in what setting should the data be gathered; from whom should it be gathered; and by whom. The decisions about these would have an impact on the description of students' language that we would be able to produce.
Qualitative research suggests the need for natural settings from which to gather data as context has an important effect on the production of linguistic data. Halliday (Halliday & Hasan, 1985) described the context of situation as being made up of what is going on, who is taking part and what role the language is playing. Changes to any of these affect perceptions of the context which then affect the language choices seen as appropriate. For example, how the teacher is perceived as interacting with students will influence students' language choices (see Khisty and Chval, 2002). For a robust model of student language to be developed from this research, it was important to keep the situation as similar as possible for all students. Yet there was a need for a variety of students to participate so that we were not relying on one or two students to provide a representative sample.

Studies into the language used by students in mathematics classrooms have, in general, only had a small number of participants. This has probably been because transcribing audiotapes of interactions takes large amounts of time (Swann, 1994) and produces huge amounts of data (Milroy, 1987 p. 22). Occasionally, studies on language in mathematics education have been done with larger numbers of participants (Rowland, 2000, Bills & Gray, 2001 and Bills, 2002). As part of his study, Rowland (2000) interviewed 230 students in one primary school to investigate their use of *hedges*. To do this he used a standardised set of questions and each interview took only five to ten minutes. The study by Bills (2002 and Bills & Gray, 2001) used 80 students who were interviewed at various times over two years. The transcribed interviews were then analysed to find the linguistic characteristics which accompanied correct calculations. Such studies constrained the language choices of students because they responded to a series of questions provided by an interviewer rather than being allowed more control over what was discussed (Rowland, 2000). However, the situations can be manipulated so that they are similar for every student and it was for this benefit that we decided to use data from the National Education Monitoring Project (NEMP). In NEMP, several hundred, randomly selected students in Year 4 and Year 8 from throughout New Zealand respond to the same set of tasks which are asked by about 100 teacher administrators (Flockton & Crooks, 1997 and Crooks & Flockton, 2001). The responses that the students give provide a snapshot each four years of what these students know in mathematics. Many of these tasks are video recorded and, therefore, can be transcribed relatively easily.

Interviewing students for NEMP is not the same as recording naturally occurring interactions in classrooms. However, the interactions between the students and the teacher administrators were similar to interactions that students would be expected to have with their own classroom teachers. Milroy (1987 p. 41), in discussing the collection of data for descriptive linguistic studies, stated that '[f]rom the interviewee's point of view, a co-operative response is often one which is maximally brief and relevant'. This could also describe the expected discourse patterns in teacher/child interactions in classrooms, except that the teacher administrators are told not to provide feedback on the correctness of the student's response, even though the provision of feedback is a typical part of classroom discourse (see Edwards & Mercer, 1987). With NEMP assessments, the students work with the same pair of teacher administrators over the course of a week and so have some opportunity to interact before doing the mathematics tasks. We were aware that the interviewer's age, gender, ethnicity and personality could affect the language choice of students (Bogdan & Biklen, 1982). On the whole, the teacher administrators –mostly female from middle-class backgrounds– would be similar to the teachers that students were likely to have in their own classrooms. Therefore, we hoped that many students would respond in the same way with the teacher administrators as they would with their classroom teachers and so would use language which closely resembled what they would use in their own classrooms. The students did the tasks in their own schools although not in their own classrooms. NEMP assessment is considered low stakes as it has no impact on the child's academic programme nor is it directly linked to school performance (Crooks & Flockton, 2001). Using the NEMP material was a compromise, as it allowed us to gather material from a large number of students where the style and set of questions were the same for all. Although it was not a classroom setting, the data was gathered in a context which was familiar to students. However, the decision to collect this data meant that we would be unable to comment on student-student interactions or even how students would use mathematics language when they had more control over the direction of the interview.

A main advantage of NEMP was that it was possible to choose students who fitted particular demographic descriptions such as gender, age (Year 4 or Year 8) and the decile ranking of the school attended. It is generally accepted in New Zealand that the decile ratings for schools relate to the socio-economic background of students (Bicknell, 1999). There are, of course, difficulties with such a categorisation as it is fraught with issues over who is making the decisions and what constitutes the factors which are relevant to such a decision (see Robinson, 1979). However, with few alternatives available, a decision was made to accept the common belief that children who came from high decile schools were from more affluent backgrounds. Tasks were also available whose responses could be related to the ethnicity of the students. These interested us as there had been studies to show that Pacific students living in New Zealand do not achieve as well as their European or Asian peers (Young-Loveridge, 2000) and so we wanted to know whether ethnicity was reflected in the language choices of students. As a result, videotapes of students were chosen based not only on gender and age, but also on their attendance at particular decile-rated schools and whether they were Pacific Islanders or not.

Tasks Selected for Analysis

To produce a rich description of students' mathematical explanations and justifications, it was necessary to look at responses to more than one task. This would enable us to see how the task as part of the context affected the language choices of students and so give us more insight into the process of making those choices. We, therefore, transcribed videos of children responding to four different tasks. From tasks done in 1997, we selected 'Better Buy' and 'Weigh Up' (see Flockton and Crooks, 1997) and 'Motorway' and 'Bank Account' from 2001 (see Crooks and Flockton, 2001). These tasks are provided in the Figures below. Instructions for the teacher administrator are given in bold.

Better Buy

Place the 100g and 50 g boxes of *Pebbles* in front of the student.
In this activity you will be using some boxes of *Pebbles*. The big box holds 100 grams of *Pebbles* and costs $1.30. The smaller box holds 50 grams of *Pebbles* and costs 60 cents.

1. Which one is better value for money?

Prompt: Which box would give you more *Pebbles* for the money?

2. Why is that box better value for money?
3. How do you know that?

Place the 20g box of *Pebbles* in front of the student.

4. This box costs 30 cents. Which is the better buy – this 20g box or this 100g?

Point to the 20 g box.

5. If I wanted 100 g of *Pebbles*, how many of these boxes would I need?
6. How did you work that out?

Figure 2.1: Instructions for teacher administrators for the Better Buy Task.

Weigh up

1. Here are four boxes of Pebbles. They look the same, but they each have a different weight or mass. Think about how you could put them in order from the lightest to the heaviest — then tell me how you would do it using the balance. Don't use the balance yet.

If the student simply says "Weigh them"...
How would they go about weighing them?

Put the placement mat in front of the student.
2. I want you to use this balance to help you work out the order of the objects, from the lightest to the heaviest. Tell me how you are working it out as you are doing it and put the boxes in order on the placement mat.

Once the student has arranged the boxes in order from lightest to heaviest, record their decisions on the recording sheet.
3. If you had to explain to someone else in your class how to work out the order from lightest to heaviest, what would you tell them to do?

Figure 2.2: Instructions for teacher administrators for the Weigh Up Task

Motorway

Show student photo.

This picture shows a busy motorway. During the day time, about 98 cars go down this road every minute.

1. About how many cars would go down the road in 9 minutes?
2. Explain to me how you got your answer.

Figure 2.3: Instructions for teacher administrators for the Motorway Task.

Bank Account

Put graph and ruler in front of student.
This graph shows someone's bank account.

Point to the words amount of money.
Up this side is the amount of money the person has.

Point to the word days.
Along the bottom are the days of a week.

Have a careful look at the graph then tell me a story to explain what is happening with the money.

Point to the beginning of the graph.

Figure 2.4: Instructions for teacher administrators for the Bank Account Task.

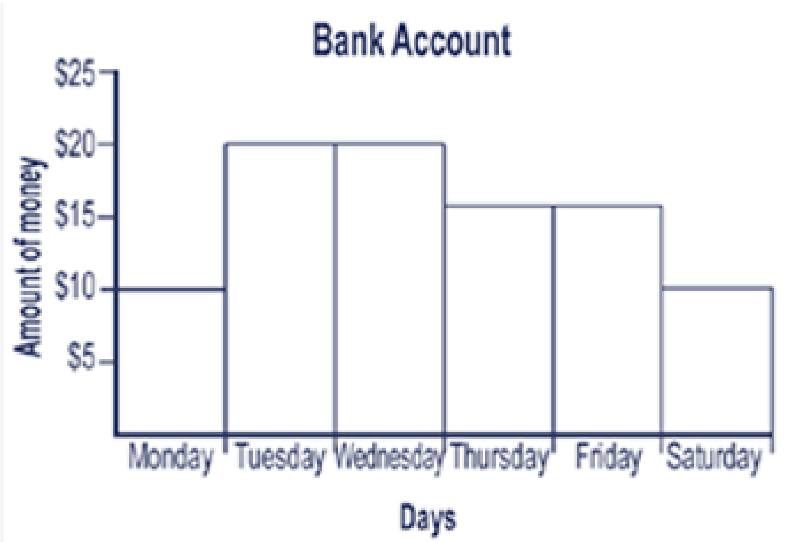


Figure 2.5: Bank Account Graph

The two tasks from 1997 were done by the same set of students whereas the ones from 2001 were done by separate groups. By using the 1997 tasks, we were thus able to see how the actual questions affected the same students' responses. Bills and Grey's (2001) research had compared students' use of linguistic features between mathematical and non-mathematical explanations. This showed that students might use certain linguistic features such as logical connectives in one context but not in another one. This suggested that context had an influence on the linguistic choices that students felt were appropriate. By being able to compare the same students giving responses to two mathematical tasks we would be able to see how much was related to the student and how much was related to the task. Students attempting the 1997 tasks could be chosen based on age, gender and decile rating of school attended whereas students doing the 2001 tasks could also be grouped according to ethnicity.

As can be seen from the instructions for the tasks, Weigh Up and Motorway required students to provide explanations of what they did whilst Better Buy and Bank Account requested justifications. Better Buy required students to justify their choice of boxes. This was often done by students making reference to the calculation that they did. Bank Account requested students tell a story about the Bank Account and we had anticipated that students would justify why the amounts on the graphs changed during the week. As the correctness of responses varied for these tasks (Flockton & Crooks, 1997 and Crooks & Flockton, 2001), it can be considered that they were mathematically challenging.

In order to develop a robust model of the language that children used in giving mathematical explanations and justifications, we needed to ensure that we had a large enough sample size. Many descriptive linguistic studies used reasonably small samples. Labov's well-known generalisations of the speech of New Yorkers was based on a sample size of only 88 speakers (Labov, 1966). Hawkins (1977) investigated the nominal groups used by five-year olds in London. Having decided on his variables of social class, IQ, gender and communication index of mothers (CI), he formed a matrix of 2 x 3 x 2 x 2 cells and assigned 5 children to each cell. However, as he did not have middle-class children who had both low IQ and low CI, two of the cells remained empty (one for boys and one for girls). This meant that he had a total of 110 samples of five-year old children's talk to analyse. Hawkins (1977) recommended using such a matrix by stating that '[i]ts advantage is that the effects of each variable may be estimated independently' (p. 8). Increasing the number of samples results in data handling issues which could make the research impractical.

Participants

It was decided for this research that a matrix of the variables of interest to us would be drawn up and we would seek to have six children allocated to each cell. This is illustrated in Table 2.1, which shows the distribution for the first two task transcripts, from 1997. For each of the four tasks, we would have a total of 72 transcripts of different children's mathematical language.

Table 2.1. ***Distribution of students chosen for analysis of Better Buy and Weigh Up, with six students per cell.***

Year	High Decile Schools		Middle Decile School		Low Decile School	
4	Girls	Boys	Girls	Boys	Girls	Boys
8	Girls	Boys	Girls	Boys	Girls	Boys

For the Motorway and Better Buy tasks, the sample of students selected was as shown in Table 2.2. These tasks, from the 2001 NEMP administration, included an enlarged sample of students from Pasifika backgrounds which we drew upon to examine whether or not there were any linguistic differences in this sub-sample.

Table 2.2. ***Distribution of students chosen for analysis of Motorway and Better Buy, with six students per cell.***

Year	High Decile Schools		Low Decile Schools non-Pasifika students		Low Decile Schools Pasifika students	
4	Girls	Boys	Girls	Boys	Girls	Boys
8	Girls	Boys	Girls	Boys	Girls	Boys

A decision was made to include those students who remained mute when asked a question as we felt that there may be patterns in their distribution. This sampling method ensured that any group, such as students from different decile schools, would contain at least 24 samples. This number increased when results across all four tasks were compared. As the students doing the two 1997 tasks were the same, our total sample size was 216. We anticipated that, with a sample size of 216 students and 288 transcripts to interrogate, patterns in linguistic choices would become apparent.

Although we would have access to the videos of students, we had to limit what we would analyse. Much information such as eye signals and gestures is lost when transcriptions are made (Swann, 1994), but, in order to keep the project manageable, it was decided that we would concentrate on the linguistic expressions used by students. As a result, only student responses were transcribed, as the NEMP procedure required the teacher administrators to ask set questions. It soon became apparent that this had not always happened, but by only having transcriptions of the student responses, we were forced to concentrate on their linguistic choices. However, it did mean that we missed opportunities to discuss the interactions between students and the teacher administrators.

One final advantage of using NEMP data was that ethics approval had already been granted. Obtaining consent from students and their parents and other interested parties such as the Ministry of Education is important in educational research to ensure that participants, especially children, are not exploited (Cameron, Frazer, Harvey, Rampton & Richardson, 1994). Yet 'going through the formal procedures that some educational systems require can be a long, laborious process' (Bogdan & Biklen, 1982, p. 122) so it was useful to have this ethical approval already obtained. Cameron et al. (1994) also raised the ethical issue of how the results would be used which might be contrary to interests of the participants. In doing research where differences between groups could be highlighted, we needed to ensure that any findings were seen as important only if they enabled teachers to better understand the mathematical learning of students. It was not possible to discover this, however, until we knew what some of these differences were.

This research set out to develop a robust model of student language in giving mathematical explanations and justifications. As a result, it was felt that a qualitative approach was the most suitable in order to achieve this. However, in determining how to implement such an approach, compromises had to be made between the aims of being as descriptive as possible, whilst also using naturalistic settings and keeping the study manageable.

Analysing the data

Once the decisions had been made about what data to collect, we then had to consider how it was to be analysed. As we were unsure what aspects of students' language were most useful in producing a robust description, it was necessary to consider how others had approached their analysis of students' mathematical language. For example, Chapman (1997) examined her transcripts to discover the ways that the teacher rephrased students' utterances and how these rephrasing were then picked up by the students. In other studies such as that by Gooding and Stacey (1993), transcripts were coded so that specific types of responses (asking questions, responses to requests for clarification) were highlighted and their use related to common attributes of the groups of students who used them. For our research, neither methods of analysis seemed appropriate as we were not focussing on the exchanges between students and the teacher administrators; rather we were comparing different students' responses.

We were also aware of the potential difficulties associated with a coding system which simply reflected what researchers expected to see (see Edwards, 1976) and how this did not support a qualitative approach to the research to be undertaken. Originally, we started by looking for particular features that we felt were more mathematical in student responses, such as the use of nominalisations or noun phrases as agents of actions rather than people, following the example of Hawkins (1977). These features were based on work described in Meaney (forthcoming).

As reported in Meaney and Irwin (2003), the results from this investigation were confusing (these are given at the beginning of each task chapter). We started again by classifying students' responses according to the clarity of their language and their accuracy. We then discussed what it was that contributed to some students' responses being classified as clear.

It was at this point that the work by Hasan (Halliday and Hasan, 1985) became valuable in our search for tools with which to do the analysis. This was because her beliefs about *text structure* enabled us to code students' responses but in ways that supported the illumination of patterns. It also enabled us to keep the context of the responses as an integral part of the description. Hasan's (Halliday and Hasan 1985, p. 56) described contextual configuration as the significant attributes of a social environment in which a text is constructed. Contextual configuration can be used to predict the text structure.

In this study, students' explanations were given in responding to questions on a mathematics task asked by a teacher administrator in a school environment. Hasan (Halliday and Hasan 1985) stated that the contextual configuration 'can predict the OBLIGATORY and the OPTIONAL elements of a text's structure as well as their SEQUENCE *vis-á-vis* each other and the possibility of their ITERATION' (p. 56 capitals and italics in original text). However, as Hasan also pointed out, the relationship between language and situation is bi-directional and some elements of the text structure will, in fact, help to construct the situation. For example, when a student provided a minimal response and the teacher administrator kept prompting, sometimes this probing became more about teaching the student than about assessing their current understanding. If these further questions become an obligatory element of the situation, then the situation changed from one of assessment to one of teaching.

By looking for the patterns of obligatory and optional elements and how they are sequenced and repeated across different students' responses, we could describe different children's perceptions of the situation. As there had been no previous work in this area, our coding was inductive rather than looking for expected elements, thus we hoped to limit the problems identified by Edwards (1976) in regard to coding.

In order to ensure reliability of results, all coding and counting was done with at least one person doing the majority of the work and another doing some checking. At times, the checking was done by research assistants who were trained in what was being counted or coded. Occasionally, the Year 4 and Year 8 data were counted or coded separately after initial joint coding or counting. There was some cross checking. Discrepancies between counters and coders were discussed and clarified.

A qualitative analysis needs analytical tools which do not constrain the data but which support the uncovering of patterns within it, so that a rich description of students' language choices can be made. Although we also began to look at the data through other lenses, Hasan's ideas about contextual configuration enabled us to reveal subtle differences in how groups of students constructed their justifications and explanations in mathematics through combining text elements. We are aware that this choice of analysis tools means that our description of students' language choices will be limited to the existence and ordering of elements within their texts.

Summary

Our research question was: What are the typical linguistic choices of different groups of students when giving mathematical explanations and justifications? In examining ways that it could be investigated, we were aware that these would have an impact on what we would ultimately be able to present as our findings. A quantitative approach might have produced a more systematic description showing how often terms and expressions were present in students' speech. Yet, it may not have enabled the context in which the students' responses were made to be an integral part of that description. We were aware that we had compromised the requirement of a natural setting for our qualitative data gathering. However, it would not have been possible to make any comparisons between the language used by different groups if the setting was not the same for all students. Our ultimate goal was to contribute to teachers' understanding of how their perception of students' knowledge about mathematics was affected by students' choice of language. It seemed valuable to produce a model which was as broad as possible and still placed importance on the context. Yet we were also aware that by looking at student-student or student-teacher interactions during learning experiences the model that we would have been able to produce would have been significantly different. To ensure that our study remained manageable we were forced to narrow our research question and then chose data gathering and data analysis methods which provided us with a robust model of students' language in giving mathematical explanations and justifications.

The following chapters provide information on each task. They begin by discussing the 1997 tasks and then provide information on the 2001 tasks. These chapters are then followed by further discussion on stories in mathematical tasks and text structures of explanations and justifications.

Chapter 3

Better Buy

Task and acceptable answers

This task came from the 1997 administration and asked students to indicate which of two boxes of Pebbles was better value for money. Results were selected from 6 boys and 6 girls of high, middle and low decile schools. There was no special Pasifika group in 1997.

Both Year 4 and Year 8 students were asked the first three questions. Year 8 students were asked the additional questions numbered 4 – 6.

Place the 100g and 50 g boxes of *Pebbles* in front of the student.
In this activity you will be using some boxes of *Pebbles*. The big box holds 100 grams of *Pebbles* and costs $1.30. The smaller box holds 50 grams of *Pebbles* and costs 60 cents.
1. Which one is better value for money?
   Prompt: Which box would give you more *Pebbles* for the money?
2. Why is that box better value for money?
3. How do you know that?

Year 8 only
Place the 20g box of *Pebbles* in front of the student.
4. This box costs 30 cents. Which is the better buy – this 20g box or this 100g?
Point to the 20 g box.
5. If I wanted 100 g of *Pebbles*, how many of these boxes would I need?
6. How did you work that out?

Figure 3.1 Instructions for the Better Buy question.

We chose to compare responses on the first three questions that were given to both age groups. Logically, this common task requires a comparison of two dimensions, weight or mass and price, and three mathematical calculations: 50g+50g=100g, 60c+60c=$1.20, and $1.20 < $1.30. This required the students to combine two values, price and weight, and select the one with the better ratio. Asking for better value implies selecting less cost for the same amount, or same cost for a greater amount.

The major difference between the subgroups that were asked this question was that only 7 of 36 Year 4 students were accurate in their reasoning for why one box was better value for money, and 27 of 36 of Year 8 students could give appropriate answers. More than half of the Year 4 students attended to only one dimension, usually price, and several of those who noted both dimensions either did not compare them or were inaccurate in the necessary calculation. Some indicated that they did not believe that value for money depended on a comparison, one saying 'because they probably wouldn't cheat on you if they were good' (Year 4 girl from a middle decile school).

Table 3.1 below shows the focus of the answers given by Year 4 students. In addition to the 21 (48%) who attended to only one element in the problem, there were 9 students who attended to both but did not have a suitable way of making a direct comparison, such as doubling the price of the 50 gram box and comparing that price to the price of the larger box which held twice the weight.

Table 3.1. ***Year 4 students responding to different elements*.***

Weight or size	Cost	Both mentioned but value not attended to	Both, giving value for money	No reason
1 HG 1 MG 1 MB 3 LBs 2 LGs	1 HB 5 HGs 1 MB 2 MGs 2 LB 3 LGs	1 HB 4 MB 2 MGs 1LB 1 LG	4 HB	1 MG
8 (22%)	12 (33%)	9 (25%)	4 (11%)	1 (3%)

* HG - girl from a high decile school
MG - girl from a middle decile school
LG - girl from a low decile school
HB - boy from a high decile school
MB - boy from a middle decile school
LB - boy from a low decile school

An answer that incorporated both weight and price but did not integrate them adequately was:

> Because it's less in money and um, like if you get them for kids to eat this is much small, this is less (Year 4 girl from a low decile school)

The concept of value for money as the result of a calculation appears to be difficult for this age group and particularly for middle and low decile students.

Both successful and unsuccessful students came from schools in all decile ranges. However, of those who were successful a higher proportion of students came from high decile schools than middle or low decile schools. The concept was considerably more familiar to students in Year 8, as shown in Table 3.2

Table 3.2. ***Year 8 students responding to different elements*.***

Size	Cost	Both mentioned but value for money not explained	Both, giving value for money	No reason
1 LG	2 LB 1 LG	1 HG 1MB 1MG 2 LG	6 HBs 5 HGs 5 MBs 5 MGs 4 LBs 2 LG	
1 (3%)	3 (8%)	5 (14%)	27 (75%)	

* code the same as for previous table

In this age range, 75% of the students gave an appropriate answer. The next highest percentage of students was those who identified both cost and size as important but did not explain clearly what gave value for money. Although correct students came from schools in all deciles, all students who attended to only one element were from low decile schools and most of students who mentioned both elements but did not clearly express value for money were from middle or low decile schools. It appeared that both cognitive and linguistic issues contributed to this.

A good example of an answer that did give value for money was:

> Fifty grams… um this one [Q] Because if you times that by two it's a hundred grams and, um, sixty plus sixty is a hundred and twenty, so that's a dollar twenty and selling that for a dollar thirty. (Year 8 girl from a middle decile school)

Linguistic elements employed

Comparison of mathematical agents and processes to non-mathematical agents and processes

Agents

As there were significant differences in the length of each response, it was not useful to look simply at the number of each expression in each category. Instead, it was decided to use a ratio of the number of mathematical terms compared to everyday terms. In the category of agent, we compared the number of times that students used the Pebble boxes (or a pronoun for them) as the main agent of a clause compared to their use of personal pronouns (including elliptical ones, as in the case of imperatives) (see Halliday, 1985). This is because, in the mathematics register, how things are named is important. In most mathematics, mathematical objects are the doers of actions rather than people. It was thought, therefore, that concentration on the boxes of Pebbles more closely resembled the mathematics register than the students describing themselves or others as the doers of the actions. The overall analysis is summarised in Tables 3.3 and 3.4.

Table 3.3 ***Ratios for the use of linguistic components by gender and year group.***

	Boys	Girls	Year 4	Year 8	Total
Mathematical object: person as agent	94:37 **2.5:1**	93:17 **5.4:1**	89:10 **8.9:1**	98:44 **2.2:1**	187:54 **3.5:1**
Mathematical process: non mathematical process	91:61 **1.5:1**	73:52 **1.4:1**	75:31 **2.4:1**	89:82 **1:1**	164:113 **1.5:1**

Table 3.4 ***Ratios for the use of linguistic components by Decile.***

	Low 4	Middle 4	High 4	Low 8	Middle 8	High 8
Mathematical object: person as agent	86:22 **3.9:1**	43:23 **1.9:1**	101:32 **3.2:1**	35:49 **0.7:1**	19:25 **0.8:1**	52:25 **2.1:1**
Mathematical process: non mathematical process	91:61 **1.5:1**	29:9 **3.2:2**	42:41 **1:1**	88:61 **1.4:1**	63:75 **1:1**	51:43 **1.2:1**

The largest differences in use of linguistic structures were between Year 4 and Year 8 students, although there were also marked differences by gender.

Because of the size of the groups analysed (only 6 students per cell), apparently large differences between the small subgroups could be caused by one or two students who had different response patterns. This was the case for the ratio of middle decile Year 4 students using a mathematical object in their clauses, in comparison to low and high decile groups.

Year 4 students used a mathematical object as an agent nine times as often as they used a person as an agent. This use was usually a reference to ‘this box’, ‘that’, or ‘it’. When only two boxes were being compared this was all the reference that was required for clarity. There were few references to ‘the bigger box’ while most were to ‘that one’ or ‘it’. Year 8 students also used a high proportion of references to the boxes as an agent using these terms, but were more likely to use a personal pronoun as the agent of their clauses.

The ratio of mathematical agent to personal pronoun as agent shows an irregular pattern across deciles, with middle decile students using a lower proportion of personal pronouns than the low and high decile students. This may have been because the middle decile students said less in response to this question than did the low and high decile students.

> This one because if you’ve got, you can get two of these for a lower price of that and you get the same amount, you can buy two of these you get a hundred grams and it’s only a dollar twenty. Q Because that you just, because if you bought two you’d times that by two, sixty cents, it’s a dollar twenty and net fifty net grams by two you get a hundred gram net (Year 8 boy attending a high decile school)

The older students who were more likely to use mathematical calculations were much more likely than the younger ones to use a number or a mathematical operation as the agent of a clause than were the younger students.

As was discussed in Chapter 1, Bills and Grey (2001) showed that students who used ‘I’ and ‘you’ in their responses were more likely to give accurate answers especially if they were also categorised as ‘generic’ or ‘general’. This was because they were not referring specifically to themselves or the teacher administrator but rather to the general actions that any person would do. Given that Year 8 students were more likely to provide an accurate response and use ‘you’ in their responses, it seemed that a similar relationship was evident in our data. Bills and Grey did suggest that ‘you’ was common classroom talk and so it was students who were comfortable with the mathematics who were more likely to use classroom ways of discussing mathematics. However, more investigation into this was needed. This is discussed further in the next section on Text Structures.

Process words

In the processes, the ratio was one between mathematical and non-mathematical verbs. Mathematical verbs showed a relationship between things or doing mathematical actions such as ‘measuring’, ‘estimating’ and quasi-mathematical actions such as ‘figuring out’. Non-mathematical verbs described other types of actions such as ‘putting on’, ‘taking off’ or personal actions such as ‘saying’, ‘thinking’, ‘having’ or static verbs such as ‘is’ in ‘this is a box’. These final verbs need to be distinguished from the relationship verbs such as ‘this is heavier’ where the ‘is’ is describing a relationship and is very typical of the mathematical register. In Better Buy, as in Weigh up, relationship was the focus of the question. Students responded by using verbs that emphasised this relationship, usually in the form of ‘it costs more’, ‘it’s heavier’, ‘it’s lighter’, or ‘it’s second lightest’.

For both age groups the second most common classification of verbs were static ones, such as ‘is’. These were used in statements such as ‘this is better’. Similarly, for both age groups mathematical verbs were the third most common category. There was a higher proportion of mathematical verbs for Year 8 students than Year 4 students. For Better Buy, the mathematical terms included 'costs', plus', 'equals', and 'times'.

In the early years of school, students are encouraged to use everyday language for mathematical terms, but by Year 4 this is expected to have been phased out with mathematical terms being used instead (Learning Media, 1992). However, these transcripts showed that students continued to use both everyday and mathematical language, and for this task, everyday language, including pronouns for agents and static verbs were more prevalent than mathematical ones.

Text Structures

Our results from counting agents and processes gave us confusing results, so we re-examined the responses and classified them according to the accuracy of the response and the clarity of the language. As can be seen in Table 3.5, the explanations that students gave in describing their reasoning for which box was the better buy do not always show that clear language is used to explain the correct answer. Nor was it clear that if you used clear language that you were likely to have the correct answer. This supports the suggestion by Ellerton and Clarkson (1996) that there is no simple relationship between good mathematics understanding and good mathematical language. However, as Table 3.5 shows, it was much more likely that students in Year 8 who answered the question correctly were able to clearly explain their reasoning.

Table 3.5. ***Clarity of explanations by accuracy of answer*.***

	Clear language	Moderately clear	Multiple reruns, vague/ elliptical
Accurate answer	2 Yr 4 HB 6 Yr 8 HBs 4 Yr 8 HG 3 Yr 8 MBs 5 Yr 8 MG 2 Yr 8 LB 1 Yr 8 LG	2 Yr 4 HBs 1 Yr 8 LB 2 Yr 8 MB 1 Yr 8 LG	1 Yr 8 MB 1 Yr 8 MG 1 Yr 8 LB 1 Yr 8 LG
Approaching accuracy- looks at both price and amount but doesn't compare	1 Yr 4 HG	1 Yr 4 HB 2 Yr 4 MBs 2 Yr 4 LGs 1 Yr 8 LG	2 Yr 4 MBs 1 Yr 4 MG 1 Yr 4LG
Doesn't compare both size and price	1 Yr 4 HB 5 Yr 4 HGs 2 Yr 4 MBs 2 Yr 4 LBs 1 Yr 8 LB	5 Yr 4 MGs 2 Yr 4 LBs 1 Yr 8 LB	1 Yr 4 MB 1 Yr 4 MG 2 Yr 4 LGs 2 Yr 4 LBs 1 Yr 8 HG 2 Yr 8 LGs

*student identification as in Table 3.1

It is clear that students who are most likely to give clear descriptions of their accurate reasoning were those students from high decile schools, although a number of students from middle decile schools were also able to do this. Students from low decile schools were less likely to have reasoned the answer correctly and to have clear language. Slightly more boys gave an accurate response than girls, whilst in the lowest level of accuracy, slightly more girls did not compare size and price than boys.

Although Table 3.5 provides some information about the differences between groups in the language they used to explain their answers, it was not sufficient to understand what were the essential parts of a clear, accurate response. It was decided therefore to investigate the features of clear language so that the typical structure of children's explanations could be identified. Using the ideas of Hasan (Halliday and Hasan, 1985), we looked for text elements and how they were combined to form a justification of their calculation.

From interrogating the data, it was clear that every student's explanation contained one or more or the following three features. These were *Premise*, *Consequence* and *Conclusion*. Premises are statements of ideas upon which the student's reasoning is built. These are the linguistic equivalences of Krummheuer's (1995) grounds which were described in Chapter 1. The students in responding to this task used two types of Premises. One was the repetition of a fact that was given in the question, such as 'It's fifty grams and that's 100 grams'. These were labelled as Factual Premises and were also seen in Motorway task. The other Premise was when a hypothetical situation was mooted, such as 'If I buy two of them …'. Descriptions of the ideas built on these Premises are labelled as consequences. For example:

Because if you buy two of these boxes,	it's going to equal a hundred grams	and only cost a dollar twenty.
Premise	Consequence	implicit Conclusion

The final feature of these explanations was a Conclusion. This is where the student made a reference to better value. Only nine students used an explicit Conclusion in their response. However, 25 other students used words such as 'more', 'only', 'but' to cue the listener to the fact that a comparison had been made. These were labelled as implicit Conclusions. Given that these were oral explanations where the context was shared between the child and the teacher administrator, it is to be expected that the listener would have to supply some background information to what they were being told (Halliday, 1985). It is perhaps more surprising that some students chose to be so explicit in their reasoning. If the Conclusion came before the Premise (and the Consequence), then the student was most likely pre-empting the question asking for their reasoning when they responded to the question about which box was better value.

The students used one of ten different combinations in giving their reasoning. Table 3.6 provides examples of each of these combinations and the number of students who used the different types of Premises. In the examples, Q stands for a question or prompt from the teacher administrator.

Table 3.6. *Text structures.*

Text structures	Examples	No. of students using hypothetical premise	No. students using factual premise
premise (1, 2, …) – consequence (1, 2, …) – conclusion	Because if you, um, if you put two of them together it will only cost, um, if you buy two of these it will cost a dollar twenty, um, and fifty times two is a hundred, and that one's, ah, ten cents more, than if you buy two of these.	5	3
premise (1, 2, …) – consequence (1, 2, …) – implicit conclusion	Because if you do fifty, if you do it's sixty cents so then you do two times sixty, and it equals one twenty and that's one thirty, and it should be one twenty.	10	5
premise (1, 2, …) – conclusion	Because, that there try, that's half the size of this one, and they charge ten cents more.	0	2
premise (1, 2, …) – implicit conclusion	Well, there's lots of pebbles in it and it costs only sixty cents.	1	8
conclusion – premise (1, 2, …) – consequence – implicit conclusion	Umm, this one's better value because if you bought two of these you'd have a hundred grams and it would only costs a dollar twenty.	3	0
implicit conclusion – premise (1, 2, …) – implicit conclusion	Just buy two of those. Q Because those are sixty, and that's a hundred, and you get ten cents off.	0	1
conclusion – premise (1, 2, …)	Probably this one here because you don't have to pay as much. Q But this one here would be the best to buy cos it has the most.	0	3
implicit conclusion – premise (1, 2, …)	Because it's only sixty cents and that's one dollar thirty.	1	2
premise (1, 2 ,…) – consequence (1, 2, ...)	Because, when you add it, sixty and sixty together which equals that it's a dollar twenty.	4	7
premise (1, 2 ,…)	Because it's a fifty gram and not a hundred gram, oh, a hundred gram and the hundred gram is a dollar thirty and the fifty gram is only sixty cents.	0	17

By looking at how text elements were combined, we hoped to better understand children's perceptions of the situation in which they were operating. It would appear that when students are required to respond to the 'why is that box better value for money?', every child provided a Premise. This is an obligatory feature of every child's response, regardless of year level, gender or decile level of their school. Optional elements were Consequences and Conclusions, as not every child included these elements. In this sample, in regard to sequencing, Premises always came before Consequences, but they were also found after or before Conclusions. Consequences, unsurprisingly, do not occur in students' explanations unless preceded by a Premise. As can be seen from Table 3.6, Consequences were more likely to occur when preceded by a hypothetical Premise. In regard to iteration, Premises and Consequences occurred repeatedly within a student's explanation, so that there have been Premise, Premise, Consequence, Premise, Consequence or Premise, Premise or Premise, Consequence, Premise, Consequence. Conclusions, whether explicit or implicit, only occurred once in any child's explanation except with one Year 4 boy who began and then ended his explanation with a Conclusion.

As can be seen from the examples given in Table 3.6, logical connectives had a role in the combination or iteration of the different elements. Bills and Grey (2001) had also found that the use of logical connectives was related to the likelihood of students providing an accurate response. From the 72 samples, when a beginning clause such as 'this box would be' is ignored, 62 students prefaced their explanations with 'because'. One began with 'so', three with 'well' and two with 'if'. Table 3.7 shows that there were no apparent patterns across groups for those who used 'because'.

Table 3.7. ***Use of 'because' by different groups.***

	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Medium	High	
'because' used as a preface to the explanation	29	33	33	29	22	19	21	62

It would certainly seem that many students believed that a response to a 'why' question should begin with a 'because' in a school setting. However, it does not appear that the children who began their answer with 'because' but then completed it by giving a repeated fact understood that this answer would be considered inadequate. The following two excerpts illustrate this:

> .....Um, because it, that box has more Pebbles than that box (Year 4 girl from a high decile school)
>
> Because it says a hundred gram, ah no, because it's, I don't know, they've just got, because it's got a hundred in it, and, ah no, because, well they're both the same really, because that's only thirty cents away from a dollar and that's only, ah, this one probably. (Year 4 girl from a middle decile school)

At a superficial level, these students appeared to have appropriate language resources. However, when contrasted with students who can use language to develop their thinking, there are significant differences. A student is more likely to be able to think through a problem if they recognise that a 'because' is needed and know that to use it appropriately requires a speaker to provide a logical reason. The following excerpt provides an instance of this:

> Q
> I think it's actually a hundred gram which, a hundred grams which costs a dollar thirty.
> Q
> Because it's got fifty more and it's exactly the same if you buy two of these.
> Q
> Because sixty plus another sixty.......ah, umm (too soft to hear).
> Q
> Yeh, I've just changed my mind.
> Q
> Yes, these Pebbles are ten cents cheaper than these Pebble*s*, because if you plussed these two Pebbles together they equal a dollar thirty, and if you plussed these two Pebbles, if you bought, if you bought these Pebbles which and you wanted to get a packet like these, this or the same amount cheaper when you could buy six, two of these which would cost less than buying these. These are ten cents less.
> Q
> I think that the fifty gram one is cheaper than the hundred gram one. (Year 4 boy from a high decile school)

The last student in the course of talking about his thinking was able to locate an error in his reasoning. The first two students gained no advantage from knowing that they were expected to use ‘because’ in order to respond to a ‘why’ question. Donaldson (1986) described three different modes for explanations: the empirical; the intentional; and the deductive. It is the last of these which is needed in logical explanations, yet it would appear that the girls were trying to provide an empirical explanation instead. It may be that in believing that they need to use such expressions, students are hindered in their mathematical understandings. By using ‘because’, students are guided into trying to give an empirical explanation rather than the logical one needed in this situation. In this case, their inadequate language is contributing to the perception that they have inadequate mathematical knowledge. It may be that students could do the mathematics to solve the problem but do not recognise it as being necessary. There are some significant differences in which groups of students provided logical reasons with their use of ‘because’, as illustrated in the following table.

Table 3.8. ***Use of ‘because’ with a logical reason following.***

	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Medium	High	
‘because’ followed by logical reason	11	15	1	25	5	8	13	26

The most significant difference in who used ‘because’ in a deductive explanation is between Year 4 and Year 8 students. Donaldson’s (1986) research suggested that ‘because’ and ‘so’ in deductive explanations were acquired at about eight years old, which is much later than their use with empirical explanations. The results in Table 3.8 suggest that most Year 4 students are unable to use ‘because’ appropriately in deductive explanations. However, age alone is not the only determiner. Twenty-one percent of students attending low decile schools followed ‘because’ with a logical reason compared with 63% of students attending high decile schools.

Other text elements were also preceded or followed by logical connectives. Hypothetical Premises are most likely preceded by ‘if’. 22 out of 23 students who used a hypothetical Premise began with ‘if’ whilst the final student began with ‘when’. The distribution of students using these logical connectives can be seen in Table 3.9. Factual Premises were not preceded by a logical connective.

Table 3.9. ***Logical connectives given before a Premise.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Medium	High	
‘when’ before a premise	1	0	0	1	0	1	0	1
‘if’ before a premise	11	11	6	16	4	9	9	22

Just as there were no gender differences in who gave a hypothetical Premise, there were also no gender differences in who used a logical connective at the start of these Premises. However, it would seem that Year 4 students attending low decile schools were the least likely to give a hypothetical Premise and use a logical connective at its start.

Table 3.10. ***Logical connectives given with Consequences.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Medium	High	
'and' before a consequence	1	1	0	2	1	0	1	2
'so' before a consequence	1	2	1	2	0	2	1	3
'and then' before a consequence	1	0	0	1	1	0	0	1
'then' before a consequence	2	0	0	2	0	1	1	2
'and' between consequences	3	6	3	6	1	4	4	9
'so' between consequences	1	2	1	2	0	1	2	3
'but' between consequences	1	0	0	1	0	1	0	1

Thirty-eight students followed a Premise with a Consequence. Of the 23 students who used Consequences which were preceded by hypothetical Premises, 22 could have begun them with a 'then' to follow the 'if' which began the Premise. However, only one student used 'then' to join the Consequence to the Premise. Table 3.10 shows the distribution of students who joined a Premise to a Consequence or a Consequence to a Consequence with a connective. When there was iteration of Premises and Consequences within the responses to this task, there was a variety of connectives which joined them together as well as instances when no connectives were used at all. Therefore, sometimes one student may have used more than one connective within their response whilst other students may have used none.

The most noticeable thing about Table 3.10 is the lack of students who used logical connectives in front of Consequences. Although 38 students used Consequences, only 7 joined Premises to Consequences with a logical connective. Of these 7 students, two used a causal connective, 'so'. More students joined Consequences together with logical connectives, but there were still only 13. Most of these used 'and' which is narrative rather than causal. When the distribution of students is considered overall, it would appear that Year 8 students are more likely to use logical connectives with Consequences. It would also seem that they are least likely to be used by students attending low decile schools. There were no differences according to gender. The number of students were small and no clear trend can be given. However, a similar pattern was evident in the logical connectives used in front of Premises

Conclusions were also often preceded by logical connectives. 34 students followed a Premise or a Consequence with a Conclusion or an implicit Conclusion. The distribution of students who used logical connectives to join these Conclusions to their preceding text elements is given in Table 3.11. In the 11 times that Conclusions (explicit and implicit) started the utterances, 'because' always followed.

Table 3.11. ***Logical connectives before Conclusions.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Medium	High	
'and' before a conclusion	9	9	7	11	5	7	6	18
'so' before a conclusion	2	4	1	5	0	3	3	6
'but' before a conclusion	2	3	2	3	0	3	2	5
'then' before a conclusion	2	0	0	2	1	1	0	2

As with the use of other logical connectives, it is Year 4 students attending low decile schools who are least likely to join Conclusions to their preceding text elements. There is no gender difference in those who used logical connectives in this situation.

This analysis presents a far more complex situation than that suggested by Bills (2002). His research had shown that, although students had been exposed to procedural and explanatory language and could use it in other situations, they did not always choose to do so in explaining how they arrived at their calculations. Our data suggests that the use of certain text elements clearly correlated with the features identified by Bills. For example, students almost always used 'you' or 'I' in a hypothetical Premise. 'You' was used by 52 students. Only six students did not use them in either a hypothetical Premise, a Consequence or a Conclusion. In all cases 'you' could have been replaced by the more formal 'one' as it was not used to refer to the teacher administrator but to a generalised person. Rowland (1995) commented on a similar use of 'you' in his research and suggested that it pointed to an expression of a generalisation. In the responses to this task, the students seemed to use it more to provide a description of the conditions under which something would be true. 'If you got two of those it will be the same as that but it would be ten cents less' enables the cost and mass of both boxes to be made equivalent, thus allowing a comparison of cost, which is a necessary part of illustrating which box is better value. This suggests that in responses to this task that 'you' was used in a very specific part, the Premise. If it is not used in the Premise, it very rarely appeared in other elements of the text structure. However, if it was used in the Premise, it was also likely to be continued to be used in the other elements found in that response. Not all groups of students used the same combinations of text elements, as can be seen in Table 3.12.

Table 3.12. ***Use of text structures by different groups.***

Text Structures	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Medium	High	
premise – consequence – conclusion	1	7	2	6	0	3	5	8
premise – consequence – implicit conclusion	10	5	2	13	3	7	5	15
premise – conclusion	1	1	0	2	2	0	0	2
premise – implicit conclusion	6	3	7	2	2	4	3	9
conclusion – premise – consequence – conclusion	1	2	1	2	0	1	2	3
implicit conclusion – premise – implicit conclusion	0	1	0	1	0	1	0	1
conclusion – premise	0	3	2	1	0	1	2	3
implicit conclusion – premise	0	3	1	2	1	1	1	3
premise – consequence	6	5	5	6	6	4	1	11
premise	11	6	16	1	10	2	5	17

There are some clear differences in which groups used which text structure. On the whole, boys in Year 8 from high decile schools were most likely to use Premise – Consequence – Conclusion structure. Year 4 girls on the other hand were most likely to just provide Premises or a Premise and a Consequence for their reasoning than boys were. Students were also likely to come from low-decile schools if they only used these text structures. Boys were much more likely to use an explicit Conclusion in their explanations than girls (13:4), but an equivalent number of boys and girls used implicit Conclusions (12:15). Year 8 students were much more likely to include a Conclusion (implicit or explicit) in their text structures than Year 4 students. However, if Year 4 students did give a Conclusion, it was more likely to be an implicit one than an explicit one. This suggests that as students get older they are more inclined to complete an explanation with a rounding off statement which links back directly to the original question. However, as was the case with logical connectives, it would also seem that decile level of school attended also affected a student's likelihood of giving a Conclusion. There also seems to be a difference in gender, with girls being less likely to give an explicit Conclusion than boys.

Text structures and clarity of language and accuracy of response

In Table 3.3, of the 29 students who gained an accurate answer, 22 were considered to have clear language. Of these, 7 used the Premise – Consequence – Conclusion structure, while a further 11 used a Premise – Consequence – implicit Conclusion structure. A further two students use a Conclusion – Premise – Consequence structure. The remaining 2 students used a Premise – Consequence combination. It would appear then that clear language which accompanies an accurate answer is most likely to be a combination of all three elements. Of the remaining 7 students who gained a correct answer, only 2 students included all three elements in their responses.

If the social environment is similar for all students, why is it that there are these differences between students in the structures of their explanations that they give? Certainly understanding and ability to solve the problem seems to allow students to make use of the Conclusion element of the text structure. In some ways, it is obvious that if a student is unable to resolve the problem then they would have nothing to conclude. However, there also seem to be differences between boys and girls in their perceptions of the explicitness of the Conclusion which is required. What makes boys chose to be more explicit than girls? This was a task in which there was a lot of shared experience between the teacher administrator and the student, yet these boys chose to be very explicit in their reasoning. We can consider the text structures that were used in explaining students' reasoning, as being on a continuum from requiring little of the listener to requiring the listener to provide a large amount of their background knowledge to the task. Students who only gave a Premise in their response require the listener to back-fill in most of the necessary information in order for the reasoning to be considered acceptable. As has been suggested elsewhere (Meaney, 2002), students' perceptions of who their audience is will have an impact on the information they provide in their responses. It may well be that students who provide very explicit responses are aware that an assessment situation requires them to presume that the listener has no prior knowledge and that they need to provide as much information as possible. Students who gave an unclear or elliptical answer may have been judged to have not solved the problem correctly even when they had chosen the 50 gram box because they could not provide a clear explanation. This has implications for teachers in regard to what they should provide in the way of modelling appropriate answers in the classroom.

Would students who knew about an expected text structure be able to use it to their advantage in helping them solve the problem? Or would a similar situation occur to that where students obviously knew that 'because' begins a response to a 'why' question but did not understand that it needed to be accompanied by a logical reason? Bills (2002) suggested that as students could use linguistic features in non-mathematical explanations, their use or non-use in mathematical explanations reflected their thinking about mathematical concepts. Our results certainly suggest that, on the whole, Year 4 students were unable to determine a successful strategy to solve this problem and providing them with a text structure for their answers may not be useful. However, some Year 8 students, by knowing about an appropriate text structure, may be able to use it to help them solve the task appropriately. Further research is needed to see whether such an intervention is beneficial.

It was also surprising to find that responses which provided a Conclusion, either explicit or implicit, were most likely to have had a hypothetical Premise, often making use of a 'you' as the doer of the action. From considering formal mathematical texts, it was expected that there would be more use of objects as the doers of the actions (see Meaney, forthcoming). Certainly, by the time that students complete high school, it is expected that they would have gained this aspect of the mathematics register. Our research shows, however, that at the end of primary school there are very few students who are clearly explaining their reasoning and who do not have a person as the agent. Yet students who have less extended responses are more likely to only provide information about objects. Is using a generic 'you' in the explanation, a necessary phase that students need to go through in order to be able to give extended explanations using objects as the agents later in their mathematical career? Further research is also needed to see when a change occurs in students' responses and whether all students go through such a sequence in the responses that they give.

Chapter 4

Weigh Up

Task and acceptable answers

There were three parts to this task. The first of these asked students to plan how to compare four boxes of different masses, the second part asked them to describe their actions as they compared them, and the third asked them how they would explain how to do this task to a peer. Each of these required students to put emphasis on different parts of their explanations and this is reflected in the majority of students' responses. The instructions for the teacher administrator were the same for Year 4 and Year 8 students. Instructions for what the teacher administrator was to do are presented in bold.

Weigh Up

Throughout this activity encourage the student to explain what they are doing and thinking about.
Place the four boxes in a row in front of the student, in order A, B, C, D.
1. Here are four boxes of Pebbles. They look the same, but they each have a different weight or mass. Think about how you could put them in order from the lightest to the heaviest — then tell me how you would do it using the balance. Don't use the balance yet.

If the student simply says "Weigh them"...
How would they go about weighing them?

Put the placement mat in front of the student.
2. I want you to use this balance to help you work out the order of the objects, from the lightest to the heaviest. Tell me how you are working it out as you are doing it and put the boxes in order on the placement mat.

Once the student has arranged the boxes in order from lightest to heaviest, record their decisions on the recording sheet.
3. If you had to explain to someone else in your class how to work out the order from lightest to heaviest, what would you tell them to do?

Figure 4.1: Instructions for the Weigh Up Task.

The first portion, labelled Plan, was scored as 'clear, logical, complete', 'nearly complete', 'on the right track but substantially incomplete', or 'other'. The second portion, labelled Description, was scored as 'all correct', 'one inversion' or 'worse'. The third portion, labelled Explanation, was scored as 'clear, logical and complete', 'nearly complete', 'on the right track but substantially incomplete', or 'other'.

Weigh Up Plan

This section looks specifically at the responses to the first question, which we have labelled as the Plan. As was the case for each task, Table 4.1 begins this section by showing which students had clear language and accurate answers. This provides a general orientation as to how students responded to the task in regard to their ability to do the mathematics and their ability to use clear language to explain their Plan. As can be seen there were many students who struggled with what this part of the task involved.

Table 4.1. ***Clarity of speech versus accuracy of answer*.***

	Clear language	Moderately clear but vague on specific details	Unclear, multiple reruns, vague	Elliptical
Accurate	1 Yr 8 HG 1 Yr 4 LG 1 Yr 4 HB	4 Yr 8 HB 1 Yr 8 MG 3 Yr 8 LG 5 Yr 8 LB 2 Yr 4 HB 3 Yr 4 HG 1 Yr 4 MG 2 Yr 4 LG 1 Yr 4 LB	1 Yr 8 HB 2 Yr 8 MG 1 Yr 8 LG 1 Yr 4 LG	1 Yr 8 MB 2 Yr 8 MG 1 Yr 4 MB 1 Yr 4 LB
Approaching accuracy	1 Yr 8 MB 1 Yr 8 HG 1 Yr 4 HG 1 Yr 8 MG	3 Yr 8 MB 2 Yr 8 HG 1 Yr 8 LG 2 Yr 4 HB 3 Yr 4 MB 1 Yr 4 MG 2 Yr 4 LG	1 Yr 8 MB 1 Yr 4 HG 1 Yr 4 MB 1 Yr 4 HB 1 Yr 4 MG 1 Yr 4 LB	1 Yr 8 HG 1 Yr 4 MG 1 Yr 8 LG
Unclear about the task		1 Yr 4 MG	1 Yr 4 MG 1 Yr 4 LB	1 Yr 8LB 2 Yr 4 LB 1 Yr 8 HB 1 Yr 8 HG 1 Yr 4 HG

*student identification as in Table 3.1

There were equal numbers of students who were judged as being accurate as there were students who were only deemed as approaching accuracy. A further nine students seemed unsure of what was involved, either by providing a minimal response or by attempting to answer a completely different question. Quite often it seemed that they were trying to ensure that two boxes equalled the weight of the other two boxes. The following comes from a Year 4 girl attending a middle decile school:

> Uh, Ok … what I'd do is I'd measure two of the Pebbles in one and if they don't equal up to the same I'd put one of the Pebbles down and get another box and put it on the weight, and if that weighed exactly right then I knew that that was the right balance.

The distribution of students does not show any major trends. Five of the six Year 8 boys attending a high decile school were judged as having a correct answer but none were considered to have a clear explanation. There was only one Year 8 girl from a high decile school who had an accurate response. There were equal numbers of Year 4 boys and girls from high decile schools who gave accurate responses. The only group of students not to be unclear about the task were boys attending a middle decile school.

By looking more carefully at the text structures which made up the students' responses, it is possible to investigate whether there was any correlation between judgements about the clarity of the explanation and their accuracy. The following section, therefore, describes the text elements and the common combinations found in these responses as well as the logical connectives joining these combinations.

Text Structures

In the Plan, like in other aspects of this task, there were many elements used in a variety of combinations. This may have been related to the extended answers required by the task. It would seem sensible for longer responses to be more likely to use a variety of elements. Yet in the Bank Account task where an extended response was often provided, there was not as great a variety of elements. It seems more likely that the variety of text elements was related to the fact that there were several ways that the Plan could be answered. Students were told that they could handle the boxes but that they were not allowed to use the balance, just describe what they would do with the balance. Some students, therefore, talked about their estimation of the order from their handling of the box, whilst others concentrated on what they would do once they were allowed to use the balance. As a result, the answers were quite varied. The following excerpts provide examples of these different types of answers.

In the first example, a Year 8 girl from a middle decile school described what she did as she weighed the boxes in her hands.

> Well, right now, by just shaking them, umm, I think there's the lightest amount of Pebbles in this box because I can hear them the most whilst they're moving quite a lot and then see, I don't think that there's that many in that box

As a response to comments and questions by the teacher, she began with a Premise which was followed by a Supposition, 'I think', with an elaboration, 'there's the lightest amount of Pebbles in this box'. This was followed by a Consequence 'because I can hear them the most' which actually preceded its Premise 'whilst they're moving quite a lot'. Another Consequence 'and then see' follows before the Supposition' I don't think' with its Elaborator 'that there's that many in that box'. Suppositions are elements that are described in greater detail below.

The second example illustrates what a Year 8 boy from a high decile school would have done with the balance:

> Umm, you'd get those two and find out the heavier one and put that like that and then test those two and the heavier one would go there and then you could test those two and the heavier one would go there and you'd keep doing it until they're all right.

This response began with a Premise which was followed by a Consequence and this combination was then repeated. The response was completed with an implicit Conclusion followed by an Elaborator.

The elements used were Premise, Consequence and Conclusions, both explicit and implicit, as there had been in the Better Buy task. But there were also Introductions, Elaborators, Suppositions and Physical Consequences. The students who used these elements is given in Table 4.3 and the elements themselves are described in the next paragraphs.

Table 4.2. ***Use of text elements by different groups.***

Text elements	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
introduction	2	1	2	1	1	1	1	3
premise	35	31	31	35	22	21	23	66
elaborator	25	21	18	28	14	16	16	46
consequence	20	26	24	22	13	16	17	46
physical consequence	11	10	11	10	10	4	7	21
conclusion	0	2	1	1	0	0	2	2
implicit conclusion	4	7	4	7	3	5	3	11
supposition	10	5	3	12	6	6	3	15

In Table 4.2, it can be seen that Premises were used by most students. However, unlike Better Buy, Motorway and Bank Account where every student used a Premise, there were six students who did not use a Premise in responding to this part of the Weigh Up task. One Year 4 boy from a low decile school did not answer at all but the remaining five students gave a response which did not include a Premise. Often these students used a Physical Consequence instead. For example, the following response is from a Year 8 boy from a high decile school:

> This one feels the lightest and then
> Q
> That's the lightest, that's the second lightest and then that one and that one

The boy began with a Physical Consequence and after the teacher's comment gave an implicit Conclusion describing the order of the boxes from lightest to heaviest.

In this section of the task, there were more Year 8 students who used Elaborators, implicit Conclusions and Suppositions than Year 4 students. More girls used Suppositions than boys. However, more boys appeared to use Consequences than girls. There did not seem to be any other major differences in the distribution of elements between the groups.

In the Motorway task, an Introduction only provided a personal response. In this task, the Introduction was an element that set up whether the student was giving a personal or generalised response. As is seen in Table 4.2, there were three students who used Introductions. Two students began their responses with 'what I'd do is' and 'what you could do is'. These both included an Elaborator embedded within the main clause. In both clauses 'is' is the main verb and 'what I'd do' and 'what you could do' are the Elaborators. The use of 'you' suggests a more generalised response than when the students uses the first person, 'I'.

The only other student who used an Introduction did not do so at the beginning of the response. This year 8 boy from a middle decile school began with an Implicit Conclusion, 'I've done it', and it was only after a prompt from the teacher administrator that the student re-started with 'I went like', which made it clear that this was a description of what this student had done rather than a generalised statement. This last example was much closer to the Introductions used in the Motorway task which were all related to the calculation being described.

Suppositions were clauses which put forth a proposition. There were several ways that students did this. Using ‘say’, ‘perhaps’, ‘probably’ or ‘you think’ seemed to set up a hypothetical situation whilst ‘I think’ was more likely to show uncertainty about the validity of a result or the suggestion being made. In this part of the task, there were 15 students who used Suppositions.

Four students used ‘say’ which was coded as a Supposition. A Year 8 boy from a low decile school used ‘say’ to make a suggestion which could then be further developed in the following utterance, ‘Um, just pick any one and put them on, and then, say that that one was heavier than this one’. A Year 8 boy from a middle decile school also used ‘say’ in the following extract from his response: ‘say A’s the next heaviest, it feels like..’. A Year 8 girl from a middle decile school also said, ‘...and then, C, say if this was the lightest then I’ll go C and D’. Another Year 8 girl from a high decile school also used ‘say’ in the following, ‘measure ah, say A and B together and see which one’s heavier’. ‘Say’ therefore was a marker to show that it was a hypothetical rather than an actual situation being described. In these cases, ‘say’ appeared to be used as an intermediate step between only talking about the specific situation in which the student is involved and being able to produce a generalisation which would cover any possibility. This similar to the ‘general’ category that Bills and Grey (2001) found in their research.

In a similar manner, a Year 8 boy from a middle decile school said, ‘if I say that that’s lightest...’. Although coded as a Premise because of it being the basis for his argument, it shared many similarities with Suppositions. There were two other suppositions which used the expression ‘you think’ which also seemed to fit this category. A Year 8 boy from a low decile school said, ‘by grabbing ... um, ah just one that you think it is’ and a Year 4 girl from a high decile school said, ‘and you’d put down the one that you thought was the heaviest’. These students were responding to the part of the question about what would you do if you were able to use the balance.

A Year 8 boy from a high decile school who had used ‘say’ in his response, also used the expression ‘perhaps B’s still heavier’ which was coded as a Premise but shared many of the characteristics of a Supposition by suggesting a possibility. This student was the only one to use ‘perhaps’ in this way. As well, there were three students who used ‘probably’ to suggest uncertainty about the Premise, Physical Consequence, Consequence or Conclusion that they were making. A Year 4 boy from a high decile school said at the end of his response, ‘like it won’t quite be down there, probably be about there’. A Year 8 girl, also from a high decile school, said in a fairly disjointed response, ‘okay, you’d probably put those two on’. The third use of probably was by a Year 8 girl at a low decile school:

> Ok, you could go like that, and that one there’s lighter than that one .. I think .. yeah, that one there’s **probably** lighter than that one, and, that’s **probably** lightest up there .. and that one there, and that one there.
> Q
> **Probably** do it the same way, just put like A there..

It is difficult to know how much this student was using ‘probably’ to show a suggestion and how much she was using it as a hedge to lessen the likelihood of losing face in front of the teacher administrator.

A similar difficulty in making judgements about the purpose of these elements also arose with students' use of 'I think'. The other nine students who used Suppositions used 'I think' to express uncertainty about the correctness of their order of the boxes after they had weighed them by hand. As was the case with 'I think' in the Motorway and Bank Account tasks, these expressions can be used as hedges to reduce the possibility of the student losing face. In these situations, the students could have been certain about the result, but did not want to appear too assertive. For example, they may have been able to guess that the next part of the task required them to actually use the balance to find a definitive answer so they did not want to pre-empt doing this. As was seen in the example of the Year 8 girl above, it is difficult to determine how much the 'I think' reflects genuine uncertainty and how much it is used to acknowledge the power relationship between the student and the teacher. This kind of difficulty is not uncommon in research on hedges (Meyerhoff, 1987). As a result, 'I think' was labelled as a Supposition rather than a hedge. The following table shows the distribution of Suppositions and other proposing devices.

Table 4.3. ***Use of supposition and other proposing devices.***

Suppositions	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
say	3	1	0	4	1	2	1	4
perhaps/ probably	2	2	1	3	1	0	3	4
you think/ thought	1	1	1	1	1	0	1	2
I think	6	3	2	7	4	4	1	9

There were far more uses of 'I think' than any other types of Suppositions. Although the numbers are small, it would seem that students from high decile schools were less likely to use 'I think'. It would also seem that girls were more likely to use Suppositions than boys and they were used predominantly by students in Year 8. This last trend is interesting because it would be anticipated that more Year 4 students than Year 8 students would have difficulty in making an accurate judgement by hand of the order of the boxes, but they are not the ones to express uncertainty about their estimation. It may be, of course, that there were fewer students in Year 4 who responded to this question by estimating the order of the boxes. However, Table 4.2 shows that equal numbers of students in Year 4 and Year 8 used a Physical Consequence to describe the results of an action that they had undertaken. This suggests that equivalent numbers of students chose to respond to this task by weighing the boxes by hand.

A new element found in the responses to this task was one which described the consequence of an action but was not related to a Premise. These were labelled as Physical Consequences and an example of one is the following from a Year 8 Pacific boy: 'and these two kind of feel the same'. Although more common in the second part of this task, there were instances in responding to the first question where students began with or only used Physical Consequences in their responses.

There were also quite varied combinations of text elements in the structures. For example, a Year 4 girl attending a high decile school said the following:

> Pick it up and if they're heavy, leave it by itself, and if the other one's are light, umm, if they are like this one
> Q
> B and, C is the heaviest and B, A and D are light and um
> Q
> You'd pick them up and feel the weight, and there's the second one …
> Q
> The two last ones down there put them in the same one, oh no, the big one, the small one goes down there and the big one, the big one and the small one goes down …

This response contains a Premise – Consequence – Premise – Physical Consequence – Premise – Consequence – Physical Consequence – Premise – Physical Consequence combination. Although some students used simple text structures, complex combinations were common and can be seen in the Table 4.4. One Year 4 boy from a low decile school only replied 'no' to all teacher prompting. The '+' indicates that the text combination could also be followed by other elements.

Table 4.4. ***Use of text structures by different groups.***

Text structures	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
intro – elaborator – premise – consequence – elaborator +	1	1	1	1	0	1	1	2
premise	4	5	5	4	3	3	3	9
premise – elaborator +, +	9	5	4	10	5	5	4	14
premise – consequence +, +	14	14	15	13	9	7	12	28
consequence – premise +, +	1	1	2	0	0	2	0	2
premise – physical consequence +, +	1	3	1	3	3	1	0	4
premise – supposition +, +	2	0	0	2	0	1	1	2
physical consequence +, +	3	3	4	2	3	1	2	6
supposition – implicit conclusion – premise – consequence – elaborator +, +	0	1	1	0	0	0	1	1
implicit conclusion	1	2	2	1	1	2	0	3

The most interesting thing about the results in Table 4.4 is the lack of any clear patterns in their distribution. This is partly because of the small numbers of students who used many of these combinations of elements. There are only three combinations which were used by more than 9 students. Of these, two had similar distributions across gender, year level and decile level of school attended. Only the text structure which began with Premise – Elaborator, with other elements following, was clearly used by more girls than boys and by Year 8 students more than Year 4 students. This pattern is emphasised when another 4 students who also used a Premise – Elaborator combination are added to these results. It is interesting to note that two Year 4 students began their responses with Consequence – Premise combination joined with 'because'. Donaldson (1986) stated that this combination was more common in her research that than ones where the action was joined to its result with a 'so'. Although very few students used 'so', as can be seen in Table 4.7, the Premise – Consequence combination was far more prevalent in responses to all tasks.

Table 4.5 sets out the combinations which included an Elaborator. The first number in each box gives the number of students using that combination with an Elaborator, whilst the second number after the '/' provides information about the total numbers of students who used the original element from Table 4.2.

Table 4.5. ***Text structures with combinations including an Elaborator.***

Text structures containing:	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
premise – elaborator	12/35	6/31	7/31	11/35	8/22	5/21	5/23	18/66
consequence – elaborator	18/20	17/26	19/24	16/22	10/13	11/16	14/17	35/46
supposition – elaborator	5/10	3/5	0/3	8/12	3/6	5/6	0/3	8/15
physical consequence – elaborator	1/11	1/11	1/12	1/10	0/10	1/5	1/7	2/22
(implicit) conclusion – elaborator	1/5	2/8	1/4	2/9	0/4	1/4	2/5	3/13

Table 4.2 shows that equivalent numbers of boys and girls used Elaborators. However, from Table 4.5 it can be seen that girls used more combinations with elaborators in them (37 to 29), with the main difference being in the use of a Premise – Elaborator combination. When comparisons are made between the initial numbers of students who used Premises, Consequences, etc and those who used these elements in combination with Elaborators, there are some major differences between groups. If girls used Consequences in their responses, they were more likely to follow them with an Elaborator than boys were. Although the numbers are small, there also seems to be a tendency for students attending middle decile schools to combine Suppositions with Elaborators than any other group. Year 8 students also seemed more likely than Year 4 students to combine a Supposition with an Elaborator. However, the number of Year 4 students who used a Supposition was very small (3) and it is difficult to know how reliable this conclusion is. What could be said is that if a Year 8 student used a Supposition then they were quite likely to follow it with an Elaborator. Students who used Physical Consequences and, to a lesser extent, implicit Conclusions were less likely to combine these elements with an Elaborator than with other text elements.

The relationship between Consequence and Elaborator elements is strongly influenced by the distribution of the Premise – Consequence – Elaborator combination. This combination was frequently used by students in their responses to all parts of the Weigh Up task. For example, a common response was something like the following which came from a Year 4 boy in a middle decile school; 'you could pick them up and see if they are heavy or light'. 'You could pick then up' is the Premise 'and see' is the Consequence with 'if they are heavy or light' as the Elaborator of the Consequence.

The way that different combinations of text elements are joined can support the cohesion of the student's text. The following sets of tables show the distribution of various combinations of elements and the logical connectives used to join them together. The first pair of tables is about the combination of Premise – Consequence (with and without an Elaborator as the next element) and Premise – Elaborator – Consequence. It looks at who used them anywhere within their responses.

Table 4.6. ***Text structures containing Premise and Consequence combinations.***

Text structures containing:	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
premise – consequence (without elaborator)	5	6	7	4	6	0	5	11
premise – consequence – elaborator	14	15	16	13	8	10	11	29
premise – elaborator – consequence	3	2	0	5	3	2	0	5

More Year 4 students than Year 8 students used a Premise – Consequence combination but only Year 8 students used a Premise – Elaborator – Consequence combination, although there were only 5 of these. None of these students were from a high decile school. Although 10 students from middle decile schools used a Premise – Consequence – Elaborator combination, none used a Premise – Consequence combination which was not followed by an Elaborator. This is interesting because in Table 4.5, the difference between the groups using a Premise – Elaborator combination within their response was not great. The numbers are small thus limiting what can be reasoned from this result.

Table 4.7. ***Logical connectives between Premise and Consequence and Premise – Elaborator and Consequence.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
then	3	4	2	5	1	4	2	7
and	14	17	19	12	10	10	11	31
and so	0	2	0	2	0	0	2	2
so	3	1	0	4	0	3	1	4
and then	3	1	2	2	2	1	1	4

It appears that there are more students using logical connectives than students using text elements, because sometimes the same student might have several combinations of Premise (– Elaborator) – Consequence within a response and have different logical connectives between elements. This can be clearly seen in the students who came from middle decile schools. This group used the least number of these text combinations, although not by a large amount, but were the group who used the most logical connectives, although once again not by a large amount.

Table 4.7 suggest that there were few uses of ‘so’ and these were all by Year 8 students. There did not seem to be any major differences between the users of other logical connectives. In this part of the Weigh Up task, it was most common for ‘and’ to join Premises, with or without Elaborators, to Consequences. It may be that boys use ‘and’ in this way slightly more than girls do and that Year 4 students do so slightly more than Year 8 students. However, the differences in the numbers are not so great as to be definitive. This use of ‘and’ would be related to the ‘put another one on and see if it’s um, different weight’, Premise – Consequence – Elaborator combination which was very common, especially amongst students from low decile schools. The following table shows what logical connectives were used between different text elements and Elaborators. This includes the use of ‘if’ in the Premise – Consequence – Elaborator combination.

Table 4.8. ***Logical connectives between text elements and Elaborators.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
consequence – if – elaborator	2	6	5	3	5	1	2	8
supposition – if – elaborator	1	0	0	1	0	1	0	1
(implicit) conclusion – until – elaborator	1	2	1	2	0	1	2	3

Table 4.8 shows clearly that there were few logical connectives which joined text elements with Elaborators. The Premise – Consequence – Elaborator combination which often used the expression ‘and see if’, as in this example from a Year 8 boy from a low decile school, ‘you hold it and see if it’s heavier’, would probably account for the 8 students who used ‘if’ between a consequence and an elaborator. The numbers are too small to determine any differences between groups.

Table 4.9. ***Logical connectives between Consequences.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
and	10	7	9	8	8	4	5	17
then	1	3	1	3	1	1	2	4
and then	8	2	3	7	2	2	6	10
so	1	0	1	0	0	0	1	1

Table 4.9 shows the logical connectives between two consequences. Once again 'and' is the most common logical connective with another ten students also using 'and then'. It would appear that students are not building up a train of reasoning, as there was only one student who used a causal connective, 'so'. In this part of the task, girls were more likely to join Consequences together with logical connectives than boys were. Students from middle decile schools were the least likely to use logical connectives. There were few differences between year levels.

There were occasions when the Consequence preceded the Premise. However, this did not occur very often. The following table provides information on the distribution of the logical connectives in this situation.

Table 4.10 ***Logical connectives between Consequence and Premise.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
because	4	2	3	3	1	3	2	6
if	1	0	0	1	0	1	0	1
whilst	1	0	0	1	0	1	0	1

Unlike the logical connectives used between Consequences, causal connectives were most evident in this situation. 'Because' was the most common logical connective, but there were only six students who used it.

Table 4.11. ***Logical connectives before a Premise.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
if	8	6	5	9	5	3	6	14
but if	1	0	0	1	1	0	0	1
so	0	1	0	1	0	1	0	1
because	1	1	2	0	2	0	0	2

Table 4.11 shows the logical connectives which preceded Premises.
In the Better Buy task, 'if' often came before the Premise – Consequence combination. This occurred in responses to this task as well. However, it is interesting to note that there were two uses of 'because' which were inappropriate before a Premise. Donaldson's (1986) suggested that students by the start of school would no longer use 'because' inappropriately. There were only two instances of this inappropriate use of 'because', but both were by Year 4 students at low decile schools. There do not seem to be any large differences between groups in using logical connectives in this position.

Table 4.12. ***Logical connectives between a Consequence and a Conclusion.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
until	1	2	1	2	0	1	2	3
and	0	1	0	1	1	0	0	1

The last place where logical connectives were often found was between the Consequence and Conclusion. Table 4.12 provides information on the very few students who chose to join Consequences to their Conclusions with a logical connective. In the Better Buy task, many more students had linked a Consequence to a Conclusion with a logical connective. In responses to this task, students did not feel that they were so necessary.

Text structures and clarity of language and accuracy of language

The differences between how students perceived what they were asked to do in this task (deciding between using their hands to order the boxes, to describing how they would use the balance to sort out the order) made determining any patterns in the responses quite difficult. As well, the small numbers of students in some groups further complicates whether there were, in fact, any trends. However, as can be seen in Table 4.13, there do seem to be some patterns in the use of text elements or combinations of elements by students perceived as having accurate or inaccurate answers relative to the clarity of their language.

Table 4.13. ***Percentage of each group of students who used particular text elements and combinations***

Description responses	Premise – Conseq	Premise – Conseq – Elaborator	Conclusion	Physical Conseq – Conseq	Supposition
Accurate and clear n = 3	100%	100%	0%	0%	66.7%
Accurate and moderately clear n = 17	88%	65%	29%	6%	24%
Accurate and elliptical n = 4	0%	0%	25%	25%	25%
Approaching accuracy and moderately clear n= 19	68%	63%	11%	5%	21%
Unclear of task and elliptical n = 5	0%	0%	0%	0%	20%

The three students who were judged to have clear language and accurate answers used Premise – Consequence – Elaborator combinations such as 'and see if'. The following was from the Year 8 girl at a high decile school: 'you'd put them in and see which one's the lighter', whilst the following was from a Year 8 girl at a middle decile school: 'I'd put one packet in, that one, and one packet in that one and I'd see which one's the heaviest'. These combinations all had 'and' as the logical connective between the Premise and the Consequence. The other student who was approaching accuracy and had clear language used a series of Premise – Consequence combinations joined by 'so' and by 'then'. This Year 8 girl from a middle decile school also used a Consequence – Premise combination which was joined with 'because'.

Students who were accurate but whose responses were considered moderately clear were slightly less likely to use a Premise – Consequence – Elaborator combination.

There was a further slight decrease in the proportion of students who were moderately clear but only approaching accuracy in their responses. This later group were also less likely to include a Conclusion than the former group. It may be that by explicitly teaching students to use a Premise – Consequence – Elaborator combination or a Conclusion that it may improve students' clarity of response but also their ability to use language as an aid to their thinking.

On the other hand, of the students who had responses judged as accurate but elliptical, none used a Premise – Consequence or Premise – Consequence – Elaborator combination. One student, Year 8 boy from a middle decile school, did join a Physical Consequence to a Consequence with 'so' but this was the only student in this group who did so. Students who were unclear about the task and elliptical similarly did not use these text elements or combinations. However, given that elliptical answers by their nature are brief, it is not surprising to find a lack of text elements or combinations.

Apart from the group that were accurate and clear, the proportion of students using Suppositions was fairly steady. It may be that having a succinct way of marking that a hypothetical situation is being described aids the chances that the response will be judged as clear. Propositional language is considered in more detail at the end of this chapter.

Compared with the analysis of the Better Buy task, the results for the Plan section of the Weigh Up task do not show very clear trends. There appeared to be little difference between the groups of students who gave answers judged on their accuracy and clarity. However, as their responses were examined, there did appear to be some differences. More girls, for example, used Elaborators, especially when combined with Premises. On the other hand, slightly more boys used consequences. Although the Better Buy task had shown that Year 8 students were more likely to be the ones who gave complex responses in terms of text structure, such a distinction was not evident in these responses. Nor did there seem to be a major distinction based on decile level of school attended and complexity of response. It would seem that what differences there were between groups were smaller and less clear cut than they had been in the Better Buy task.

Weigh Up Description

For this part of the Weigh Up task, students were asked to talk through what they were doing as they were doing it. For some students, this was difficult and five students did not use a complete clause, even though they were asked (sometimes several times) by the teacher administrator to describe what they were doing. Other students who did respond only did so because they were prompted many times.

An accurate answer was considered to be one in which the student successfully ordered the boxes from lightest to heaviest. An example of this would be the following, which came from a Year 8 girl from a low decile school:

> Ok then, I'm going to estimate it first, and I'm estimating B then A then C then D. Ok, put these two on, and see which one is heaviest out of both of them, and it's A, and I'll put A on the side, this one will be the lightest, then I'll, I'll um, I'm guessing that um, that D is heavier than, I mean C is heavier, heavier than D, and it is, so D will go there and C will go there, and I'll just check, just better check .. ooh, hang on .. and that's my estimate.

Q
Test it one more time?

Q
Oh, hang on, I'll just check it one more time...Yep C, then A .. yes (very quietly to herself)..oh, hang on, B got beat then, D, B, A and C

Table 4.14 provides information on which students had accurate answers and clear language. Half of the boys from high decile schools were able to provide a clear accurate response to this task. Of the four Year 4 students who provided a clear, accurate response only one was not a boy from a high decile school. One third of girls from low decile schools also gave a clear accurate response, but these students were all in Year 8. The students who were unclear about the task were all in Year 4. Three were from low decile schools and three were boys. Of the 12 students who were approaching accuracy and moderately clear, eight were in Year 4.

Table 4.14. ***Clarity of language vs correctness of description of what they did*****.**

	Clear language	Moderately clear but vague on specific details	Unclear, multiple reruns, vague	Elliptical
Accurate Answer	4 Yr 8 LGs 1 Yr 8 LB 1 Yr 8 MG 1 Yr 8 MB 3 Yr 8 HGs 3 Yr 8 HBs 3 Yr 4 HBs 1 Yr 4 MG	1 Yr 8 LG 2 Yr 8 MGs 1 Yr 8 HG 1 Yr 8 HB 2 Yr 8 MBs 1 Yr 8 LB 1 Yr 4 LG 3 Yr 4 MGs 1 Yr 4 MB 1 Yr 4 HB	1 Yr 8 LB 1 Yr 8 HG 1 Yr 4 LG 2 Yr 4 HGs 3 Yr 4 LBs	2 Yr 8 LBs 2 Yr 8 MGs 1 Yr 8 MB 1 Yr 4 LG 1 Yr 4 MB
Approaching accuracy	2 Yr 8 MBs 1 Yr 4 HG	1 Yr 8 LB 1 Yr 8 MG 2 Yr 8 HBs 2 Yr 4 LGs 1 Yr 4 LB 1 Yr 4 MB 2 Yr 4 MGs 2 Yr 4 HGs	1 Yr 4 HB	1 Yr 4 HB
Unclear about the task			1 Yr 4 LG 1 Yr 4 MB	2 Yr 4 LBs

*student identification as in Table 3.1

There were some differences with Table 4.1 which showed the corresponding distribution for the Plan section of this task. There are significantly more students than in the Plan section who were considered to have provided an accurate response, with many of these being perceived as having clear language. As this task was done whilst actually doing the necessary comparisons on the balance, to be considered clear did not require students to be as explicit in their response as they needed to be in the first part of the task. Below is an example of an accurate response with clear language from a Year 4 girl from a middle decile school.

Well, you put them like A and B on each one, and A's gone down lower so it's heavier than B, so I'll put that there, and then you get.. like A and C, and C's heavier, because it went down, and then you get A and D, A's heavier, so that one's there, and I'll go .. B and C, and C's heavier, and then you go D and C, and C's still heavier, and then see what the heaviest out of B and D, and B's the heaviest, and then D's the lightest.

If this is compared with a response from another Year 4 girl from a middle decile school, the difference in clarity is quite obvious.

Ok .. ok .. that'll be, no I don't know that, that's one of the heavy ones and that's one of the lightest ones, and I'll try that one and that one .. this one's heavier .. that will probably be the heaviest .. and that one will be heavy .. ok, I'll take this one here off, put that one to there .. ok, that would be the next heaviest .. and .. this would be the lightest .. there

Both students worked through the comparisons to gain an accurate result and both explanations make sense when the video of the student doing the task is watched. It would be interesting to understand what prompts some students to provide explanations closer to a written version so that they can be understood without the context being known. Heath's (1982) work suggested that the students' understandings, gleaned from their backgrounds, of what types of responses 'school questions' require, are not always appropriate. Certainly it would seem that 9 out of the 17 students who were deemed to be accurate and clear came from high decile schools. Research by Zevenbergen (2001) suggested that students from middle class backgrounds are more likely to have responded at home to questions similar to those used at schools. When compared with the responses to the Plan section of the task, it seems that students found it easier to provide the extra details needed to make the explanation understandable when they were actually performing the task. This suggests that not just students' backgrounds but also the task requirements affect students' abilities to show what they know in school-expected ways.

Text Structures

In this part of the Weigh Up task, students continued to use a variety of combinations of elements from Introduction, Premise, Consequence, Supposition, Physical Consequence, Conclusion (Implicit and Explicit) and Elaborator. Compared to the Plan, there were many more students using Physical Consequences. This is not surprising, given that they were asked to talk about what they were doing. The following table shows the distribution of students using individual elements.

Table 4.15. ***Use of text elements by different groups.***

Text elements	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
introduction	2	1	3	0	3	0	0	3
premise	27	27	25	29	16	17	21	54
elaborator	18	20	17	21	11	10	17	39
consequence	25	25	22	28	15	16	19	50
physical consequence	31	32	31	32	20	21	22	63
conclusion	2	1	1	2	1	1	1	3
implicit conclusion	5	6	5	6	4	3	4	11
supposition	9	5	6	8	1	8	5	14

The most used element in these responses was not the Premise as was the case in all the other tasks and the other parts of this task, it was, instead, the Physical

Consequence. Given that there were 5 students who did not respond verbally to this task, there were only 3 students who did not use a Physical Consequence in their response. The other elements were used in similar amounts to those in the Plan section of this task.

As also was the case in the Plan, there were very few patterns in the distribution of who used which elements. Premises and Elaborators were used slightly more often by students from high decile schools. It also appears that more Year 8 students than Year 4 students used Consequences. Suppositions seem to be used more by girls than boys and by students attending middle decile schools than other types of schools. However, the differences in all of these trends are not large and so nothing can be stated definitively.

Only one student, a Year 4 girl from a middle decile school used ‘say’ as a Supposition. She said, ‘oh whether the say, I got put those two in, I’d leave that one there’, in a fairly incoherent response. There were also two instances of students using ‘say’ which were coded as other elements but had features of suppositions. A Year 8 boy from a high decile schools said, ‘and, and then you say, well, A and B and see which is the heaviest’. It is not absolutely clear what he meant, but there is a sense that he is proposing that A and B boxes should be tried next. A Year 4 boy, also from a high decile school said, ‘and just measure this B one again, that one goes D and B then I’d say A and, oops, C’, where ‘I’d say’ was coded as a Consequence but does exhibit features of being a proposition, as it suggests that A follows D and B (the correct ordering of the boxes was B, A, D and C).

Although there was a limited use of ‘say’, other terms used to suggest propositions or courses of action were coded as Suppositions. One student used ‘suppose’ to suggest that she was putting forward a proposition. A Year 8 girl from a middle decile school said, ‘yeah, A’s the heaviest with that one ... yeah, still too light, so ... suppose it’s D ... this’s about the same ... no, this one’s heavier’. ‘Suppose’ has been used to suggest that box D be tried with Box A to find out which box is heaviest and she then discovers by using the balance that D is the heavier of the two. A Year 4 girl, also from a middle decile school, used ‘will probably’ to suggest what might happen when two boxes went on the balance in the following extract: ‘put the D there, the D will probably go up, but it went down then’. A Year 8 girl from a high decile school also used ‘probably’ to lessen the certainty of her suggestions. These were not generalisations, but were rather predictions of immediate actions.

There were 11 students who used ‘I think’ in their response. It was once again, difficult to determine whether these were hedges or whether they were genuinely expressing uncertainty about the suggestion that they were making. The following is an example of a use of ‘I think’ by a Year 8 boy from a high decile school, which appears to genuinely be suggesting a proposition; ‘I think this one’s going to be the lightest because it was higher than the rest.’ But as he completes the sentence, his own belief about the certainty of the statement seems clearer, as he gives the reason for that the belief. It may well be that by the end of the sentence the ‘I think’ represents a hedge but when the boy began the sentence it was proposing an idea rather than a certainty and thus was a true supposition. This difficulty in determining students’ intentions when they used ‘I think’ has resulted in them being classified as Suppositions rather than as hedges.

As well as using 'I think' to make suggestions, there were also students who talked about 'guessing' or 'estimating' which box was the heavier of two or the order of all four boxes. The following example from a Year 8 girl from a low decile school illustrates this: 'I'm going to estimate it first, and I'm estimating B then A then C then D.' Although these were usually coded as Premises, they had many common features with Suppositions, as they were putting forward an idea as a possibility or probability rather than as a stated fact. Table 4.16 sets out the distribution of these Suppositions.

Table 4.16. ***Use of Supposition by different groups.***

Suppositions	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
say	1	0	1	0	0	1	0	1
suppose	1	0	0	1	0	1	0	1
probably	2	0	1	1	0	1	1	2
I think	6	5	5	6	2	4	5	11
Other proposing devices	3	2	1	4	1	1	3	5

It would seem that students from low decile schools were less likely to use Suppositions. The numbers are small but it may be that girls were more likely to use Suppositions than boys and that Year 8 students were more likely than Year 4 students. This matches the results from the Plan section of this task, but it may be that it is the same students who use Suppositions in both parts of this task.

The text elements given in Table 4.15 were used in a variety of different combinations. Although these combinations are similar to the ones used in the first part of the task, the distribution of students using them is not. The following table provides information on the distribution of students using combinations of text elements.

Table 4.17. ***Use of text structures by different groups.***

Text structures	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
none	3	2	3	2	3	1	1	5
intro, +, +	2	1	3	0	3	0	0	3
premise	0	1	1	0	0	1	0	1
premise – elaborator +, +	3	2	1	4	0	3	2	5
premise – consequence – elaborator, +, +	5	9	7	7	4	2	8	14
premise – consequence + other elements	1	1	1	1	1	1	0	2
premise – physical consequence +, +	13	10	12	11	8	6	9	23
premise – supposition +, +	2	1	3	0	0	2	1	3
physical consequence	1	4	3	2	3	2	0	5
physical consequence – elaborator, +, +	2	0	0	2	1	0	1	2
physical consequence – premise, +, +	1	2	1	2	2	1	0	3
physical consequence – consequence +, +	2	3	1	4	0	4	1	5
physical consequence – supposition +, +	1	0	1	0	0	0	1	1

In this part of the Weigh Up task, all students began their responses with an Introduction, a Premise or a Physical Consequence. Although most students (48) began with a Premise, 16 began with a Physical Consequence. This was more than double the number who began this way in responding to the Plan. From this larger number, it can be seen that for this part of the task, students from high decile schools were the least likely to use a Physical Consequence at the beginning of their responses. Apart from this, it would appear that the distribution of students using text structures beginning with different combinations of elements is fairly evenly spread.

As Elaborators were used by 40 students, it was interesting to look at which elements were combined with Elaborators by which groups of students. In Table 4.18, the second number is the total number of students, who used the elements: Premise; Consequence; Suppositions; and Physical Consequences.

Table 4.18. ***Text structures with combinations including an elaborator.***

Text structures containing:	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
premise – elaborator	9/27	5/27	6/25	7/29	5/16	5/17	4/21	14/54
consequence - elaborator	12/25	14/25	13/22	13/28	9/15	4/16	13/19	26/50
supposition – elaborator	4/9	2/4	2/6	4/7	1/1	3/8	2/4	6/13
physical consequence – elaborator	6/32	2/31	4/32	4/31	2/20	1/21	5/22	8/63

It can be seen from Table 4.18 that Consequences were the most likely element to be combined with Elaborators. Physical Consequences were the elements which were the least likely to have Elaborators following them. However, students were less inclined to follow a Consequence with an Elaborator than they had been in responding to the Plan. In the Plan, just over 75% of students who used a Consequence combined it with an Elaborator. In this section, half of the students combined Consequences with Elaborators. It would also seem that students from low decile schools were more likely, if they used a Consequence to follow it with an Elaborator whereas students from middle decile schools were the least likely. Except for Consequences, students from low decile schools were the least likely group to use Elaborators. However, the numbers of students are still small and it is difficult to make any conclusive comments. Apart from Consequences, girls were more inclined to follow a text element with an Elaborator than boys were. Table 4.15 showed the equal numbers of boys and girls used Elaborators, but it would appear that girls use them more often and with a wider range of text elements.

The following sets of tables show the distribution of logical connectives between different elements. Although it would have been useful to be able to present the same set of tables as was provided in the Plan part of the task, there were significant differences in where different logical connectives were used. For example, in front of Premises, ten different logical connectives were used. None of these connectives had been used by more than two students. Therefore, these results are not given in a table format.

Although there were more students using Consequences in this part of the Weigh Up task, fewer students used logical connectives to join Premises with Consequences. The first table provides information on the distribution on students using a Premise (– Elaborator) – Consequence combination of elements.

Table 4.19. ***Text structures containing Premise - Consequence combinations.***

Text structures containing:	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
premise – consequence (without elaborator)	10	4	6	8	6	5	3	14
premise – consequence – elaborator	8	11	10	9	8	2	9	19
premise – elaborator – consequence	3	0	2	1	1	2	0	3
premise – elaborator – consequence – elaborator	1	1	1	1	0	1	1	2

From Table 4.19, it would seem that girls more than boys used more of these combinations, except that boys were slightly more likely to use a Premise – Consequence – Elaborator combination than girls. Students at middle decile schools were less likely to use these structures than students attending other schools. There are also differences in the distributions of these combinations compared with those used in the first part of this task. More students, mainly girls, used a Premise – Consequence combination in this part of the task, but less students, mainly from middle decile schools, used Premise – Consequence – Elaborator combinations.

Table 4.20. ***Logical connectives between Premise (- elaborator) - Consequence (- elaborator).***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
then	1	0	1	0	1	0	0	1
and	7	4	4	7	4	1	6	11
and + if, then	2	1	1	2	1	1	1	3
so	3	4	1	6	3	3	1	7
until	2	1	2	1	1	1	1	3

The above table shows the distribution of logical connectives between premises (– Elaborators) and Consequences (– Elaborators). Although there were similar numbers of students using the combinations in the first and second sections of this task, there was significantly less use of logical connectives. For example, there were 20 fewer students using ‘and’ as a logical connective. Only the number of students using ‘so’ increased, but not by much.

This decrease in the number of connectives used was not evident when the Consequence was preceded by the Premise. There were 10 students rather than the 8 students in the earlier section of the task who joined these with logical connectives. ‘Because’, however, was the only connective used. Out of these 10 students, 8 girls used it, 8 Year 8 students and 5 students from high decile schools. When these results are combined with the results from the responses to the Plan, it would certainly seem that girls are more likely to use a connective between a Consequence – Premise combination. The trend is not as strong, but it would also appear that Year 8 students are also more likely to use a logical connective in this combination. The situation for the effect of decile level of school attended is not so clear.

Table 4.21. ***Logical connectives between Consequences.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
and	15	7	6	16	6	7	9	22
then	3	2	3	2	1	0	4	5
and then	9	3	5	7	3	3	6	12
so	5	5	5	5	3	2	5	10
because	1	1	1	1	0	1	1	2
and so	0	1	0	1	0	0	1	1

Table 4.21 provides information on the distribution of logical connectives between Consequences; 'and' is again the most common one. There are more students, mainly girls and Year 8 students, using it than had been the case in the Plan section of the task. There are similar numbers and distribution of students using 'and then' to those in the first part of the task. However, in this part of the task, more students have chosen to use 'so' between their Consequences, suggesting that they were building onto each other rather than being independent Consequences of the same Premise or Physical Consequence. The following from a Year 4 boy from a high decile school illustrates this.

> Well, I take pebble cases B and C, I put C on and see that it weighs it all the way down, **so** I put B on.

The first two clauses were coded as Premises (Well, I take pebble cases B and C, I put C on) and the next clause was a Consequence followed by an Elaborator (see that it weighs it all the way down) which was then followed by another Consequence that was introduced with 'so'. The whole of this student's response shows how he used 'so' to link together not just Consequence to Consequence but also Physical Consequence to Consequence and Consequence to implicit Conclusion.

> Well, I take pebble cases B and C, I put C on and see that it weighs it all the way down, **so** I put B on, and from what I see here, I shall take them off and then I shall put them down at the same time and I see there's a .. ah huh! B is heavier than C, I mean C is heavier than B, yeh. **So**, I don't know where they go **so** I'll, I just put them in the middle at the moment and I take C out and do it against D, see which is heaviest .. C is heavier than D....now .. C is still heavier than A **so** C is heavier than all, **so** I'll take them off .. C is the heaviest. Now I weigh these two .. will that differ? .. let's see which is heaviest...B is heavy **so** I weigh it against A .. now it's A is heavier than B **so** I put A here because it's heavier than B and B is heavier than D **so** I've got them in order

Physical Consequences as the most common text element in this section of the task were also connected to other text elements with logical connectives. The next series of tables shows the distribution of students using logical connectives with Physical Consequences.

Table 4.22. ***Logical connectives before Physical Consequences.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
and	15	7	7	15	9	5	8	22
so	4	7	7	4	4	5	2	11
because	0	1	1	0	1	0	0	1
but	2	0	2	0	0	1	1	2

Table 4.22 shows that once again ‘and’ was the logical connectives used by the most students. However, twice as many girls to boys and twice as many Year 8 students to Year 4 students chose to join Physical Consequences in this way. On the other hand, more boys than girls and more Year 4 than Year 8 students chose ‘so’ as a logical connective between Physical Consequences. However, the numbers are quite small for ‘so’, ‘because’ and ‘but’ and no clear conclusions can be formed.

Table 4.23. ***Logical connectives between Physical Consequences.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
and	7	1	6	2	2	6	0	8
but	1	1	1	1	1	0	1	2

Although Table 4.23 shows that more girls than boys and more Year 4 students than Year 8 students used ‘and’ between Physical Consequences, the numbers of students is small. Given that, in this task, there were 62 students who used Physical Consequences, it was expected that there would be many logical connectives joining Physical Consequences to preceding elements and to other Physical Consequences. Even when all the logical connectives in the last two tables are added together, it does not equal the number of students using Physical Consequences (46 to 62). This, of course, does not take into account that there may be some duplication of students in different rows of the tables.

When the results from these tables are compared with the two previous ones showing the logical connectives in front of and between Consequences, there are some differences. More students used logical connectives between Premises and Consequences and between Consequences than they did between Physical Consequences. This is perhaps not surprising, given that Consequences build on previously given information and so there is more need of a link. On the other hand, Physical Consequences reflect the results of actions and are thus less likely to be joined to anything else. However, when Consequences arose from Physical Consequences, they were often joined with a logical connective.

Table 4.24. ***Distribution of students using a Physical Consequence – Consequence combination.***

Text structure	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
physical consequence – consequence	18	18	13	23	9	12	15	36

Table 4.24 shows which students used a Physical Consequence – Consequence combination. Although there is no difference in the use of this combination by gender, more Year 8 students were likely to use this combination than Year 4 students and more students attending high decile schools were likely to use this combination than students attending low decile schools. The logical connectives joining this combination are presented in the following table.

Table 4.25. ***Logical connectives between Physical Consequence - Consequence.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
so	17	8	7	18	4	11	10	25
so then	2	1	1	2	0	2	1	3
and	5	2	3	4	3	1	3	7
and then	1	1	1	1	0	1	1	2
then	1	1	1	1	1	0	1	2

'So' is used by more students to join a Physical Consequence to a Consequence. All but one girl who used this combination of text elements used a 'so' as a logical connective. Although they used an equal number of these combinations, boys consistently used fewer logical connectives than girls. Although Year 8 students used this combination more often than Year 4 students, they also used more logical connectives proportionally than did the Year 4 students. All but one student from a middle decile school who used this combination also connected it with a 'so'. This was proportionally more than for students attending high or low decile schools.

Why is there a change in the logical connectives used in this particular combination? It is difficult to know and difficult to investigate. However, it may be that the students chose to use 'so', with its meaning of 'as a result', to emphasise the logical link between an action and the consequence drawn as a result of the action. For example, a Year 8 girl from a low decile school said this in her response:

> I'm guessing that um, that D is heavier than, I mean C is heavier, heavier than D, and it is, so D will go there and C will go there

'And' connects a Premise to the Physical Consequence but 'so' is chosen by the student to connect the Physical Consequence to its Consequence. The 'and' could be left out and the meaning would not change very much but if the 'so' was removed the link between the ideas is not transparent.

In looking at the overall picture of who used which logical connectives where in the Description responses, it would seem that girls and Year 8 students were more likely to use them to connect different text elements together. This is different to the situation in the Plan where the distribution was much more evenly spread.

Text structures and clarity of language

In order to determine the relationship between using particular text structures and being judged on accuracy of response and clarity of language, there was a need to have large enough groups for meaningful comments to be made. Unfortunately the distribution of students within Table 4.1 did not match the distribution of students in Table 4.14. In the Description responses, the only groups with large enough numbers to comment upon were those deemed to have an accurate clear response, those who were accurate but elliptical and those who were moderately clear and approaching accuracy.

It would appear that certain text elements and combinations of elements occur more frequently when students gave clear accurate answers and less frequently when giving elliptical but accurate answers or moderately clear answers which were approaching accuracy. These are the use of Conclusions (implicit or explicit), Premise – Consequence – Elaborator and Physical Consequence – Consequence combinations.

Table 4.26. ***Percentage of each group of students who used particular text elements and combinations.***

Description responses	Premise – Conseq	Premise – Conseq – Elaborator	Conclusion	Physical Conseq – Conseq	Supposition
Accurate and clear n = 17	53%	41%	35%	94%	6%
Accurate and elliptical n = 7	0%	0%	0%	0%	14%
Approaching Accuracy and moderately clear n = 12	25%	8%	17%	58%	25%

Table 4.26 provides details of which students were deemed to have accurate and clear responses and their use of various combinations of text elements. Of the 17 students, who were deemed to have accurate answers and clear language, nine included a Premise – Consequence combination with a further two students using a Consequence – Premise combination. There were only two cases when the Premise – Consequence combination was not followed by an Elaborator. Six students, five of which were in Year 8, also used a Conclusion (implicit or explicit). However, a more common combination was that of Physical Consequence – Consequence with 16 of the 17 students using this combination. ‘So’ was the most common logical connective between these elements.

Only seven students, with five being in Year 8, were deemed to have elliptical but accurate answers. Of these, none contained a Premise – Consequence (– Elaborator) combination, a Conclusion or a Physical Consequence – Consequence combination. This is not particularly significant, as in being labelled as elliptical, these students were unlikely to give extended answers. Two students, in fact, said nothing while all except one of the other students used at least a Physical Consequence. These students were unwilling or unable to discuss what they were doing while they were doing it.

Table 4.26 also provides information on the text elements used by the twelve students who were deemed as having a moderately clear response which was approaching accuracy. It can be seen that the proportion of students using these text elements or combinations is significantly lower than was for the case for students who were accurate with clear language. It may well be that, if students are explicitly taught to include these combinations in their response, they would be more likely to be able to use language to support their thinking.

It would seem that the use of Premise – Consequence (and to a lesser degree combined with an Elaborator) and Physical Consequence – Consequence combinations support providing an accurate, clear response. To a lesser extent, the inclusion of a Conclusion may also increase the likelihood that the response would be judged clear. Although the use of Premise – Consequence (– Elaborator) combination was equally useful in the Plan responses, the other combinations were not so. This reinforces the belief that students need to know how to combine different text elements but also need an awareness of the appropriate situations in which these combinations should be used.

The Description part of the task produced responses which were more likely to revolve around Physical Consequences than Premises. As such, there were differences in the use of text elements generally and in regard to which responses were most likely to be considered accurate and clear. The high use of Physical Consequences also affected the proportion of students using logical connectives and the likelihood of them being used in different positions.

Weigh Up Explanation

In this part of the task, students were asked how they would explain this task to a class-mate. On the whole, students were more expansive in their explanations than when they had responded in the Plan section, where they had to describe what they would do before doing it. NEMP's report on this task stated that '[e]xplanations improved a little after the students did the task' (Flockton & Crooks, 1997, p. 38). As was the case in the previous sections of this task, Table 4.27 is provided to give background about students' accuracy and clarity.

For this part of Weigh Up task, it was not possible to transcribe the responses of six Year 4 boys from middle decile schools. This was because, in the sample which were kept by NEMP, there was only 5 students with these demographics who were asked all three questions. As a result the total sample size is 71. As well, one Year 4 boy from a low decile school did not say anything except 'Umm.... Q No' when asked the question.

Table 4.27. ***Clarity of language vs correctness of explanation******.**

	Clear language	Moderately clear but vague on specific details	Unclear, multiple reruns, vague	Elliptical
Accurate Answer	2 Yr 8 LG 1 Yr 8 MG 2 Yr 8 HBs 1 Yr 4 HB	3 Yr 8 LGs 3 Yr 8 LBs 3 Yr 8 MGs 1 Yr 8 MB 3 Yr 8 HGs 2 Yr 8 HBs 2 Yr 4 LGs 1 Yr 4 MG 1 Yr 8 MB 1 Yr 4 HG 2 Yr 8 HBs	1 Yr 8 LG 2 Yr 8 MGs 1 Yr 8 HG 4 Yr 8 MBs 3 Yr 8 MBs 1 Yr 8 HB 1 Yr 4 MG 2 Yr 4 MBs	
Approaching accuracy	1 Yr 8 MB 1 Yr 8 HG 1 Yr 4 LG 1 Yr 4 LB 3 Yr 4 MGs 2 Yr 4 HGs	1 Yr 8 HGs 1 Yr 8 HB 1 Yr 4 LB 1 Yr 4 MB 1 Yr 4 MG 3 Yr 4 HGs 1 Yr 4 HB	2 Yr 4 LGs 2 Yr 4 LBs 1 Yr 4 MG 1 Yr 4 HG 1 Yr 4 HB	1 Yr 4 LB
Unclear about the task				1 Yr 4 LB

*student identification as in Table 3.1

Although the NEMP report suggested improvement in students' responses in this part of the task, Table 4.27 shows that it was still difficult for many students. Although more students were considered to have accurate responses, very few were deemed to have clear language. This is probably due to the high linguistic demands placed on students. In this part of the task, they had to describe a complex set of instructions for a peer and make distinctions between the four boxes. The most successful were those who were able to make general rather than specific references to the boxes. The following is from a Year 8 girl from a low decile school who was judged as using clear language and providing an accurate answer:

> Um, to weigh two of them, and then take leave .. leave the lighter one, no take the lighter one off .. and, no take both of them off and then put on the other two on, and then take the two heaviest from both of those different ones, put them on and find out which is the heaviest, then put it down the end, and then measure between the two middle ones and find out what was the heaviest, and then put the heaviest, the next heaviest on the next bit, then figure out between the lightest and the other one, like from D and B, figure out which was the lightest from them and then put the lightest there and the next one there.

Although there were some false starts, it is quite possible to follow her explanation as she makes comparisons between the different boxes. It is only towards the end that she resorts to using box labels to make suggestions about what should be done. The references to the placement mat, 'put the lightest there and the next one there' make sense when watching the video and are entirely appropriate when the teacher administrator and the students are sitting in front of the boxes and the place mat. There are some non-standard terms such as 'heaviest', which was used in 'then take the two heaviest from both of those different ones', which in Standard New Zealand English should have been 'heavier' as it is a referred to a comparison between two boxes. Many students used 'heavier' and 'heaviest' to refer to a comparison between two boxes, although ‘heavier’ was never used to refer to a comparison of more than two items.

The following example also comes from another Year 8 girl attending a low decile school. It was judged as being accurate but elliptical.

> Put one in and then, and then weigh them all, all of them to see if that’s the lightest, and then just do it to all of them.

There is much that the listener is required to fill in. Although the student is correct in the first step, the suggestion ‘then just do it to all of them’, would not enable another person to actually complete the task themselves if they had not seen it being done. However, given that the teacher administrator has just watched this student successfully complete the task, the student is aware that the teacher administrator knows what is involved in this instruction. Heath’s (1982) work on adult-child questioning interaction patterns suggests that middle class children are the most likely to provide more detail in their responses in a classroom setting when there is considerable shared knowledge between participants. Children of lower class families are less likely to fulfil the school game of providing details that the other participants in the interaction already know. The results in Table 4.27 suggest that this is not the case in New Zealand. Girls attending low decile schools were as likely as boys from high decile schools to provide a clear or moderately clear and accurate response. It would also seem that students from middle decile schools were more likely to provide a vague but accurate response, regardless of gender.

Although there were equivalent proportions of students getting accurate answers with clear language and with vague language, it would also seem that Year 4 students who provided an accurate answer were more likely to be moderately clear. This also suggests that the linguistic demands of this part of the task were demanding for these students. Some ability to use language was necessary to gain an accurate answer, but to give a clear response required more language ability than most Year 4 students had.

Text Structures

Although the same range of text elements (Introductions, Premises, Consequences, Conclusions –implicit/explicit, Elaborators and Suppositions) were used in this part, all texts began with an Introduction (only one Introduction was used) or a Premise and most responses combined at least three of these elements. Table 4.28 provides information on the distribution of students using the different elements.

Table 4.28. ***Use of text elements by different groups.***

Text elements	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
introduction	0	1	1	0	0	0	1	1
premise	36	34	34	36	23	23	24	70
elaborator	34	30	30	34	18	22	24	64
consequence	35	31	32	34	21	22	23	66
physical consequence	6	1	3	4	2	4	1	7
conclusion	1	0	0	1	1	0	0	1
implicit conclusion	15	10	11	14	7	9	9	25
supposition	7	6	4	9	2	6	5	13

All students who made a verbal response included a Premise and almost all students also used an Elaborator and a Consequence. However, the number of students using a Physical Consequence significantly dropped from the earlier parts of the task. Only one of these 7 students was a boy. Most students did choose to describe through words rather than actions what another student would have to do to solve the task. Occasionally, some students resorted to again comparing the boxes on the balance so that their Explanation resembled their Description. The following comes from a Year 8 girl at a middle decile school and gives both her Description and her Explanation.

> **Description**
> Okay, I felt that one was the heaviest so I put that in there and ... that one. …
> Q
> So, right now, I've seen that that one is really heavy, so, because like how much it takes down and that one didn't move nearly at all and this one moved a bit more than that. So that one is the lightest, definitely and B will be the second lightest and … A is next and C comes last because C was the heaviest.
>
> **Explanation**
> I'd take, I'd tell them to take which one they thought was the heaviest, say that was C, to put it in and that would touch the ground. Well, to as much as it would go and then take the, and see which one never, like, that one goes down quite a bit and so it's quite heavy and find out which one doesn't move really, it, which one moves the least. See this one barely moves it at all and this one moves it a bit more and if this one. If this one makes this one, makes it heavier then see that one is a bit heavier and so we know that D is the heaviest and B was lighter than A because it stayed up more and D was the lightest

Although the student started her explanation by just talking about what another student would have to do, in order to clarify her explanation, she began to use the balance again. As a result, there is a close resemblance between her Description and Explanation. Hence, Physical Consequences such as 'this one moves it a bit more' appeared in her response.

The following comes from a Year 8 boy at a middle decile school. It shows considerable difference between the Description and the Explanation responses. Although there is still some reference to the actual situation, as in 'put them on there', none of the Consequences in the Explanation are drawn from using the balance.

Description
Ok....well, A's still heavier....C, A..B, D.

Explanation
I'd say, ah, use your hands to find the two heaviest, then measure them then put .. then, get the heaviest one put it at the end, and then put the second heaviest one after it, then get the lightest one, then get the other two and put them on there and just find which one's heaviest from there .. oh, and with the heaviest ones, you put them on the scales as well to find out which one is the heaviest.

Although there were only 7 students who gave Physical Consequences in their responses, it is interesting to find that 6 of these were girls. More research needs to be done to see whether, on the whole, girls are more inclined to discuss what they are actually doing than to abstractly talk about instructions for another person.

As can be seen in Table 4.29, similar numbers of students used Suppositions in this part of the task as they had done in the other two sections. In the previous two parts of this task, more girls had used Suppositions or other propositional devices than boys. In this part of the task, there is no distinction between girls and boys. However, it would seem that Year 8 students used Suppositions more than Year 4 students. This was most evident, in this part of the task, in the use of 'you think'. Across the three sections of this task, it would seem that being able to use linguistic features to set up a proposition is something which students learn as they get older. As with other features like the Premise – Consequence – Conclusion combination in responses to the Better Buy task, the decile level of school attended also has an affect on the likelihood of the use of Suppositions. Students attending low decile schools are less likely to use them than other groups.

Table 4.29. ***Use of Suppositions by different groups.***

Suppositions	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
say	4	1	3	2	0	2	3	5
imagine	0	1	1	0	0	0	1	1
guess/estimate	1	1	0	2	1	0	1	2
you think	3	5	1	7	1	4	3	8
they think/ thought	1	1	1	1	1	1	0	2

As well, compared to the Suppositions used in the previous two sections, the expressions used to give propositional status to a clause were different. Whereas previously 'I think' had been the most common expression used, there were no examples of this in this part of the response. However, 'you think' or 'they think' was used by similar numbers of students as 'I think' had been in the earlier two sections of this task. Given that this task specifically asks students about what they would tell another student, it is not surprising that first person references were replaced. What is interesting is that students chose to use a second person pronoun rather than a third person one. The following example comes from a Year 8 girl from a middle decile school: 'until you think you know which one's lightest and heaviest'. The use of 'you' is unlikely to refer to the teacher administrator but to a generic 'you' as was the case in the Better Buy task. It, however, may be that students are rehearsing what they would say to another child rather than giving a generic response.

In the example, ‘you think’ suggests that the doer of the actions does not need to be absolutely certain of the lightest and heaviest box. The following example, also from a Year 8 girl from a middle decile school, does not provide this uncertainty: ‘and just do that with each one until, eventually you’ve got it figured out’, although the ‘eventually’ does suggest that the process might take some time. The numbers are fairly small, but it would seem that Year 8 students were most likely to use ‘you think’ and students from low decile schools were least likely.

It was also interesting to note that there were no uses of ‘perhaps’ or ‘probably’, but a student used ‘imagine’ in a similar manner to ‘say’, thus suggesting a proposition . ‘Guess’ and ‘estimate’ were each used by a student. Although these were not coded as Suppositions but rather as Premises, there is a sense that the doer of the action does not have to be correct.

Table 4.30 provides information of the most common combination of elements which were used at the beginning of student responses. This table shows that Premise, Consequence and Elaborator were used in different combinations at the beginning of almost all responses.

Table 4.30. ***Use of text structures by different groups.***

Text structures	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
none	0	1	1	0	1	0	0	1
intro +	0	1	1	0	0	0	1	1
premise	0	1	1	0	2	0	0	2
premise – elaborator – premise +, +	8	7	8	7	6	4	5	15
premise – elaborator – consequence +	12	11	8	15	6	10	7	23
premise – elaborator – supposition +	2	3	2	3	2	3	0	5
premise – consequence	0	2	1	1	2	0	0	2
premise – consequence – premise	1	0	1	0	0	1	0	1
premise – consequence – elaborator	10	9	11	8	6	3	10	19
premise – consequence – supposition	1	0	0	1	0	0	1	1
premise – supposition +	1	0	0	1	0	1	0	1
premise – implicit conclusion	1	0	0	1	0	0	1	1

The three combinations which were used by most students were Premise - Elaborator – Premise, Premise – Elaborator – Consequence and Premise – Consequence – Elaborator. 42 students out of the possible 71 students gave responses which began with Premise – Elaborator and were completed with other elements. A further 22 students gave responses which began with Premise – Consequence and were completed with other elements. Although it appears that students attending middle decile schools are less likely to use this last combination than students attending high decile schools, there do not appear to be any other distinctions between groups. When the combination of elements is examined more carefully, twice as many Year 8 students as Year 4 students used the Premise – Elaborator – Consequence combination. However, the most notable feature of the results in Table 4.30 are the lack of differences in groups using various combinations.

Table 4.31. ***Text structures with combinations including an Elaborator.***

Text structures containing:	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
premise – elaborator	24/36	21/34	18/34	27/36	16/23	16/23	13/24	45/70
consequence – elaborator	29/34	25/30	25/30	29/34	18/18	16/22	20/24	54/64
supposition – elaborator	5/7	2/6	3/4	4/9	0/2	3/6	4/5	7/13
physical consequence – elaborator	2/6	0/1	1/3	1/4	1/2	1/4	0/1	2/7
implicit conclusion – elaborator	3/15	3/10	4/11	2/14	3/7	2/9	1/9	6/25

As Elaborators were once again a feature of most students' responses, Table 4.31 provides information on the distribution of students using different elements with Elaborators. In the Explanation, there were significantly more students who combined an Elaborator with a Premise. In the Plan, the percentage of students using Premise – Elaborator combinations compared to those using just Premises was 27% and for the Description part of the task, it was 26%. However, in the Explanation, this rose to 45%. This increase was because 75% of Year 8 students who used a Premise combined it with an Elaborator. This increase in the number of students using this combination is probably related to the opening Premise which was often something like 'You'd tell them to use one of these' or 'Well to check first of all to find out which is the heaviest one'. As the students were asked 'If you had to explain to someone else in your class how to work out the order from lightest to heaviest, what would you tell them to do?' both Introductions and these opening Premises respond directly to the question by setting the scene for the explanation to come. However, unlike an Introduction, these Premises describe a specific action, such as telling or checking. This is different to 'What you do is' which was the only clause coded as an Introduction in this part of the task. There were only two students who did not begin their responses with Premise – Elaborator but who incorporated this combination in their responses later. This emphasises just how prevalent this combination was at the beginning of the responses. The research done by Hass and Wepman (1974) suggested that older children would incorporate more 'embeddedness' into their responses. However, it is only in the Premise – Elaborator combination in this part of the task that there appears to be a difference based on age. Their research analysed samples of students' oral stories based on picture cards. It may be that narratives, which are what the children produced in the Hass and Wepman (1974) study, provided fewer possibilities for using a Consequence – Elaborator combination but encouraged more elaboration around Premises.

For this part of the task, not only did more students use a Consequence within their response but they were more likely to combine them with Elaborators. 84% of students who used a Consequence combined it with an Elaborator at least once within their response. All students attending a low decile school, if they used a Consequence, combined it with an Elaborator. Although the differences are not great, it would also seem that, as was the case in the Plan and in the Description, students attending middle decile schools were the least likely to combine a Consequence with an Elaborator.

Students who used implicit Conclusions were less likely to combine them with Elaborators than with any other text element. There do not appear to be any other major distinctions between groups using elements with Elaborators.

It would seem that the elaboration seen in the responses to this part of the task was due to the increased use of Elaborators with Premises and Consequences. This element enables students to keep the main thread of their argument clear with the Premise – Consequence combination but give essential details through the addition of Elaborators with either of these elements. It may be that Elaborators can be considered part of Krummheuer's (1995) backings as they provide details about the constraints on ideas.

Other information which provides details of the conditions under which claims are valid comes in the use of causal connectives such as 'if' and 'so'. The following set of tables looks at the likelihood of students using particular sets of combinations and the logical connectives used between them.

Table 4.32. ***Premise and Consequence combinations.***

Text structures containing:	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
premise – consequence (without elaborator)	11	13	14	10	4	12	8	24
premise – consequence – elaborator	18	20	19	19	13	10	15	38
premise – elaborator – consequence	6	3	4	5	3	3	3	9
premise – elaborator – consequence – elaborator	8	8	6	10	4	8	4	16

Table 4.32 shows that there were no major differences between groups based on gender or age. There are some minor differences according to decile level of school attended, with students at middle decile schools more likely than others to just use a Premise – Consequence combination.

Table 4.33. ***Logical connectives between Premise and Consequence and Premise – Elaborator and Consequence.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
then	3	8	2	9	2	6	3	11
and	24	12	21	15	10	13	13	36
and then	12	12	8	16	8	6	10	24
so	1	0	0	1	0	0	1	1
because	0	1	0	1	0	1	0	1
but	0	1	0	1	0	0	1	1

The logical connectives which join Premises with Consequences provide information on how the students saw the relationship between these two elements. This is shown in Table 4.33. 'And', 'and then' and 'then' were used by substantially more students than any other logical connective. Donaldson (1986) suggested that although primary school students often use 'and' to join many ideas together in a one, long sentence, their spoken language used a far greater range of connectives. However, the results in Table 4.33 repute this, as it shows that a very limited range of logical connectives were used by the large numbers of students who used a Premise and a Consequence combination, often more than once in their responses. Unsworth (2001) stated that 'Consequential and Theoretical Explanations obviously entail greater use of conjunctions indicating cause while conjunctions in Sequential Explanations are predominantly those of temporal relations' (p. 136). Although it would be expected that most explanations in mathematics would emphasise causality, over all three parts of the Weigh Up task there was limited use of logical connectives showing causality, such as 'so'. In this part of the task, there was only one student who used 'so' between the Premise and the Consequence, but there was also a student who used 'because' inappropriately. The results in Table 4.33 suggest that most students used one logical connective consistently throughout their response. If some students had chosen to use two, then it would mean that an equal number of students did not use any logical connection in this situation. The use of narrative (temporal) connectives rather than causal connectives suggests that students perceived these explanations as being about listing the order of what they had to do rather than giving information about why they are doing it. The situation for logical connectives used between Consequences is very similar and is shown in Table 4.34.

Table 4.34. ***Logical connectives between Consequences.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
and	19	21	16	24	11	14	15	40
then	10	4	6	8	3	8	3	14
and then	17	15	12	20	7	10	15	32
so	2	3	4	1	2	1	2	5
and so	0	1	0	1	0	0	1	1
but	2	1	3	0	1	1	1	3
or	1	0	1	0	1	0	0	1
until	1	0	0	1	1	0	0	1

Students used a greater range of logical connectives between Consequences than between Premises and Consequences. However, students still chose most often to use 'and', 'and then' and 'then', which did not provide information on causality. There are some differences between the groups in who uses logical connectives between Consequences. Between Premises and Consequences, 'and' is used by substantially more girls than boys but this is not the case between Consequences. Although boys used slightly more logical connectives between Premise – Consequence combinations, girls used more between Consequences. Year 8 students also used more between Consequences than between Premises and Consequences. In both Premise - Consequence and Consequence - Consequence combinations, fewer students from low decile schools used logical connectives.

Table 4.35. ***Logical connectives between Physical Consequence and Consequence.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
and	3	0	1	2	1	2	0	3
so	4	0	1	3	1	2	1	4

Table 4.35 shows the attributes of the few students who used a logical connective between a Physical Consequence and a Consequence. Although there were many fewer Physical Consequences used in this part of the task, it is interesting to see what logical connectives were used to join them to Consequences. As there was only one boy who used a Physical Consequence in his response, it is not surprising to find that it is only girls who used logical connectives in this situation. The numbers are small and so it is not possible to draw any conclusions from this table.

Table 4.36. ***Logical connectives between Consequence and Premise.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
because	5	2	4	3	1	4	2	7
if	2	0	1	1	0	1	1	2
when	0	1	1	0	0	0	1	1

Table 4.36 provides information on the distribution of students using logical connectives within a Consequence – Premise combination. ‘Because’ remains the logical connective used by most students to join a Consequence to its subsequent, related Premise. As was the case in the previous two parts of this task, more girls chose to join these elements together with a connective than boys and students from low decile schools were the least likely to join them with a connective. This result contradicts Donaldson’s (1986) findings, in which she found that children in natural talk used an Effect – Cause combination joined with ‘because’ more often than a Cause – Effect combination joined with ‘so’. Although ‘so’ was not common, students in responding to all the mathematical task were more likely to use a Premise – Consequence combination than vice versa.

It may be that logical connectives illustrating causality are provided in other parts of the text structures. For example, logical connectives such as ‘if’ can be used before a Premise to provide the connection between it and the following Consequence, rather than having a logical connective in between. As discussed in Chapter 1, Esty (1992) stated that ‘“if … then” and “if and only if” are absolutely fundamental to mathematics’ (p. 40). It is, therefore, not so surprising to find that ‘if’ carries the major load in showing the causal relationship between ideas in mathematical explanations. This is because these connectives allow distinctions to be made on the truth quality of the mathematical statements. The following table provides information on the distribution of students using logical connectives before Premises.

Table 4.37. ***Logical connectives before a Premise.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
if	11	14	12	13	6	10	9	25
when	0	1	1	0	0	0	1	1
so	1	0	0	1	0	1	0	1
as	0	1	0	1	0	1	0	1

In the Description part of this task, 13 students had used a logical connective before a Premise, but only ‘and then’ was used by more than one student (it was used twice). The situation is very different in this part of the task. In the Explanation, Table 4.37 shows that ‘if’ was used by over a quarter of the students to join a Premise to a Consequence. However, there do not appear to be any major differences between the groups of students using logical connectives before Premises. It seems that for this part of the Weigh Up task, that causality is shown in the logical connectives used before Premises rather than between Premises and Consequences.

In the Plan responses, many students used a Premise – Consequence – Elaborator, often using the phrase ‘see if’ as the beginning of the Consequence – Elaborator. In the Description responses, where students performed actions rather than describing what they might do, this phrase was not common. However, it reappeared in responses to the last part of the Weigh Up task, as in the example from a Year 8 girl from a low decile school, ‘put another one in and see if it’s um heavier or not’. Table 4.32 showed the distribution of the 38 students who used the Premise – Consequence

– Elaborator combination. The following table provides information on the distribution of students using logical connectives in front of elaborators.

Table 4.38. ***Logical connectives before an Elaborator.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
if	4	5	3	6	1	5	3	9
as	1	0	0	1	1	0	0	1
but	0	1	1	0	0	1	0	1
or	2	0	2	0	1	1	0	2
because	1	0	1	0	1	0	0	1

The number of students using logical connectives before Elaborators is similar to those in the Plan responses. However, in this part of the task, there were many more students using a Consequence – Elaborator combination, so that, proportionally, Consequence – if – Elaborator was not used as often in this part of the task. If students did not use a logical connective before an Elaborator, the Elaborator would often be a relative clause such as ‘which one’s lightest’ as in the following example from a Year 4 girl from a high decile school: ‘and see which one’s lightest’.

The other place where logical connectives were commonly used in other tasks was between a Consequence and a Conclusion (implicit or explicit). In this part of the task, there were 26 students who used a Conclusion. This is double the number of students who used this element in the Plan part of the task and significantly more than the three students who used it in the Description part of the task.

Table 4.39. ***Logical connectives between Consequence – Conclusion.***

Logical connectives	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
so	3	1	3	1	1	2	1	4
and	1	4	2	3	2	2	1	5
and then	8	1	2	7	4	1	4	9
then	1	1	2	0	1	0	1	2
until/till	2	1	1	2	0	3	0	3

A total of 23 students used logical connectives between a Consequence and a Conclusion. As most students only used one Conclusion in their responses, it seems that Consequences are almost always connected to Conclusions with a logical connective. As girls and Year 8 students used Conclusions more often, it is not surprising to find that they also used more logical connectives in the Consequence – Conclusion combination.

In this part of the Weigh Up task, girls and Year 8 students were more likely to use logical connectives than any other groups. Students from high decile schools were slightly more likely to use logical connectives than students attending other decile level schools. There appears to be no difference in the use of logical connectives which were causal rather than narrative.

Text Structures and Clarity of Language

Although more students were considered to have provided an accurate answer, not many were thought to have clear language. However, the numbers of students who were unclear about the task or who gave elliptical responses were greatly reduced from the previous two sections of this task. Differences in the number of students in groups means that a direct comparison of text structures between the sections of this task is not possible.

Table 4.40. ***Percentage of each group of students who used particular text elements and combinations.***

Description responses	Premise – Conseq	Premise – Conseq – Elaborator	Premise – Elaborator - Conseq	Concl	Physical Conseq _ Conseq	Suppo
Accurate and clear n = 6	83%	67%	17%	17%	17%	33%
Accurate and moderately clear n = 22	82%	64%	45%	50%	9%	27%
Accurate but vague n = 15	80%	53%	33%	33%	6%	20%
Approaching accuracy and clear n = 9	67%	44%	44%	22%	0%	11%
Approaching accuracy and moderately clear n = 9	67%	55%	44%	55%	11%	0%
Approaching accuracy and vague n = 7	100%	71%	14%	14%	0%	14%

Table 4.40 shows the proportion of students in each group who used particular text elements or combinations. If students were accurate, regardless of their language fluency, they were more likely to use a Premise – Consequence combination than if they were only approaching accuracy. However, use of a Premise – Consequence – Elaborator or a Premise – Elaborator – Consequence – Elaborator combination does not seem to be related to the accuracy or the clarity of the response.

Conclusions were most likely to be used when students were approaching accuracy in their responses. This is not surprising, as students who did not provide complete details of each step would often finish their explanations with a broad statement such as ‘so on like that’ as in ‘… if B’s lightest then it will be the second to lightest, and so on like that, until you think you know which one’s lightest and heaviest’. Given the shared experience from doing the earlier parts of this task, this is an entirely sensible approach by students in responding to this part of the task.

As there was a limited number of Physical Consequences used in this part of the task, they do not appear to have any influence on accuracy or clarity of responses. On the other hand, it does appear that students who gave accurate answers were more likely to use Suppositions than those who were only approaching accuracy. This is interesting and suggests that students who were able to put forward propositions were more likely to be able to cope with hypothetically determining the order of the boxes.

As was the case with the Description responses, it would seem that the combination of Premise – Consequence (with, to a lesser extent, an Elaborator) was more likely to be used by students providing an accurate response. It would also appear that Premise – Elaborator – Consequence combination was used by students who were deemed as giving moderately clear responses, regardless of accuracy. However, Physical Consequence – Conclusion combinations which had been commonly used by students providing accurate answer in the responses to the Description part of the task did not have the same significance in the Explanation section. This is not surprising given the decreased use of Physical Consequences in responses to this part of the task.

What does the Weigh Up task as a whole tell us?
The Weigh Up task required students to give three different, mathematical explanations. Although there were some overlaps in how students structured their responses, there were also some differences. This section describes the differences and similarities between these explanations. Some of the trends evident across all parts of the task included the position of Elaborators in the text structures and who used them most frequently, as well as the use of Physical Consequences, which were text elements not found in the responses to other mathematics tasks. This section also looks at the most frequent combinations of text elements, such as Premise – Consequence – Elaborator and the use and frequency of different logical connectives. It finishes by commenting on the ways that students dealt with *possible* actions or events that arose in the Plan and the Explanation parts of this task. Being able to deal with possibilities is important, as it relates to the early stages of providing generalisations in mathematical explanations.

Elaborators are text elements which provide further information about the element in which it is embedded or which precedes it. Elaborators were mostly relative clauses which began with 'which' or 'that' or a subordinate clause, such as one which began with 'if'. In this task, Consequences are the text elements that are most likely to be followed by an Elaborator. Physical Consequences and Conclusions are the elements which are the least likely to be used with an Elaborator. This can be seen in Table 4.41.

Table 4.41. ***Percentage of students using text elements who then followed them with Elaborators.***

	Plan	Description	Explanation
premise – elaborator	27%	26%	64%
consequence – elaborator	77%	52%	84%
supposition – elaborator	53%	46%	53%
physical consequence – elaborator	9%	13%	28%
conclusion (implicit) – elaborator	15%	0%	24%

In the responses to the Explanation section, which were the longest ones, more students used more elements followed by Elaborators that either of the other two responses. However, most of this increase was as a result of more students using Premises with Elaborators at the beginning of their responses.

In Tables 4.5, 4.18 and 4.31, there were no significant differences in the numbers of students who used Elaborators according to gender. However, as evident in Table 4.42, once the distribution of students who used Elaborators with different text elements was examined, some differences became apparent.

Table 4.42. ***Percentage of students using text elements who then followed them with Elaborators.***

	Plan		Description		Explanation	
	Girls	Boys	Girls	Boys	Girls	Boys
premise – elaborator	34%	10%	33%	19%	67%	62%
consequence – elaborator	90%	65%	48%	56%	85%	69%
supposition – elaborator	50%	60%	44%	22%	71%	33%
physical consequence – elaborator	9%	9%	18%	6%	33%	0%
conclusion (implicit) – elaborator	20%	25%	0%	0%	20%	30%

Table 4.42 shows that, on the whole, combinations with elaborators were more likely to be used by girls than by boys. This was particularly so in the combinations of an Elaborator with a Premise or a Physical Consequence. However, Elaborators with Conclusions were most likely to be used by boys, although the numbers of students who used this combination were quite small. The distribution of Elaborators used with Consequences and Suppositions was dependent on the section of the task. When boys did use proportionally more Elaborators with other elements than girls, the differences were not as great as when girls used more Elaborators than boys.

The use of Physical Consequences is, unsurprisingly, closely related to when students were actually performing actions. This shows up in the large numbers of students who used Physical Consequences in their response to the Plan and to the Description parts of this task. There is also a relationship between the combination of Physical Consequence and Conclusion and clarity of language in the responses to the Description. 94% of students who were deemed to have given an accurate and clear response to this task included this combination in their response.

As was discussed in Chapter 1, Bills (2002) suggested that particular words in students' descriptions of mental computations correlated with students providing accurate answers. In considering the results from the three parts of this task, it can be said that certain combinations of text elements are also related to students providing accurate responses. The Premise – Consequence – Elaborator was used by a significant number of students in responding to each part of the task. In the responses to the Plan, 40% of students used this combination. In the Description part of the task, 26% of students used it and in the Explanation part of the task 53% of students used it. Students who were considered to be accurate were more likely to use this combination than students who were only approaching accuracy. Although Tables

4.13, 4.26 and 4.40 suggest that Premise – Consequence combinations were important, they also show that it was more likely that these combinations were followed by an Elaborator than not.

As well as the Premise – Consequence – Elaborator combination being important, in the Description part of the task, students who gave accurate and clear responses almost all used a Physical Consequence – Consequence combination. In mathematical explanations, when physical actions are needed, it would benefit students if they learnt to use this combination both to support the clarity of their language but also to help their thinking. This is because they move from just describing the consequences of their actions to drawing a logical consequence from the results of that action.

There also seemed to be some distinct trends in the use of logical connectives across the different parts of this task, which are related to how different text combinations were used. Certainly when the Physical Consequence – Consequence combination was used in the Description responses, more often than not this was joined with 'so' suggesting a causal relationship between the ideas expressed in the Physical Consequence and the Consequence.

Causality is not shown through logical connectives joining Premises to Consequences as had been the case in some of the other tasks. Girls and Year 8 students were the most likely to use a logical connective in this position, with 'and' being the most common logical connective. Instead, causality was marked through the use of 'if' in front of a Premise or an Elaborator in the Premise – Consequence – Elaborator combination. If 'if' was not used, then the causality had to be implied from the relative clause which was the Elaborator. For example, in 'take two and measure them first, and see which one's the heaviest, and then put the heaviest one up on the heaviest end', 'which one's the heaviest' is the relative clause which was coded as an Elaborator. It presumes that the outcome of finding which box is the heavier is easily done and so it can be incorporated into the following suggestion which was 'and then put the heaviest one up on the heaviest end'. Thus the causality is between the proposed action and what follows. Contrast this with 'then weigh them all, all of them to see if that's the lightest, and then just do it to all of them'. In this case the 'if' at the beginning of the Elaborator marks this causal relationship in a more pronounced way. However, more has to be presumed from the original action for the final result to be valid. Both sorts of Elaborators allow students to deal with possibilities in a succinct way so that further actions can then be described. It would seem that students used the Premise – Consequence – Elaborator combination as one way of dealing with possibilities.

In order to discuss in a clear and concise manner the possible ordering of the four boxes, students needed ways of talking about possibilities. This has many links to the ability to use generalisations, which are important aspects of mathematical explanations and justifications. In order to interpret and use them correctly, students need to know the constraints on an equation being true.

In this task, both the Plan and the Explanation sections required students to describe general ways of ordering the boxes. Anthony and Walshaw (2002) suggested that there was a need 'to discern generality in students' informal utterances' and to understand '[t]he interplay between generalisation and justification' (p. 52). By analysing their responses, it is possible to understand how typical primary students limited their generalisations so that they remained true. The next few paragraphs describe first the Premise – Consequence – Elaborator combination and how the logical connectives and other aspects of this combination limit when the possibilities were true. In many cases, this combination of text elements could be considered as examples of Bills and Grey's (2001) 'general' responses, as they provided a rule which would cover most instances. Then Suppositions, especially students' use of 'say' could be related to Bills and Grey's 'generic' responses, as they were used to provide examples of how these rules operated. Students who were only able to talk about the specific boxes in front of them would be considered as giving 'particular' responses. From Tables 4.13, 4.26, 4.40, there would seem to be a relationship between the use of Premise – Consequence – Elaborator combinations and/or the use of Suppositions with the responses which were deemed accurate and clear.

Tables 4.13, 4.26 and 4.40 show that more students used the Premise – Consequence – Elaborator combination when giving their Explanations, with the least number of students doing so when giving their responses to the Description part of the task. This supports the belief that students used this combination so that they could talk about the possibilities in ordering the boxes. The Premise – Consequence – Elaborator allows an action to be proposed and the consequent result of that action to be carried forward into the step, even though the result may not be definite. As was described earlier, sometimes a logical connective such as 'if' is used to mark the conditions under which the following action would be true. However, a relative clause, such as 'which is heavier', was also used where the conditions for the statement to be considered true are within the object itself.

Students also used Suppositions, such as 'say', to mark a possibility. Although the number of students using Suppositions was similar, the expressions used did differ across each section of Weigh Up. In responses to the Plan part, four students used 'say'. In their responses to the Description part only one student used 'say', whilst in their Explanation responses, five students used 'say'. By using 'say', students marked the fact that it was a possibility rather than a fact, as in the following: 'so say it was D, you put it in front of it'. 'Suppose' and 'imagine' were more Standard English words which also indicate that what was following was a proposition rather than a fact. However, only two students throughout the task used these expressions. It is interesting to note that these expressions which mark possibilities are verbs.

As well as verbs, adverbs such as 'perhaps' and 'probably' were also used occasionally to indicate a possibility. Although four students used these expressions in their responses to the Plan and another two students used them in their responses to the Description, no student used them in their responses to the Explanation. This would suggest that these adverbs have only a limited role in describing possibilities. However, when the actual examples such as the one from a Year 8 girl from a high decile school are examined this is not the case. The example was 'say B's the heaviest so A's lighter, I'd do that later, I'd perhaps put D, perhaps B's still heavier, so throw out D, then C, perhaps C's heavier'. Here 'perhaps' could be replaced by 'say' and the meaning would remain the same.

The large numbers of students who used 'you think' in the responses to the Explanation seemed to have used them like 'say'. This can be seen in the following example from a Year 8 boy from a high decile school: 'See how heavy it is in your hands first and then you put the two that you think are the lightest in'. Once again an action is suggested and the consequent result, although not a certainty, is carried forward into the following action. Although 'think' as a verb is also used to mark a possibility rather than an actual fact, the inclusion of the pronoun suggests that a person, 'you' could actually change the certainty of the actual result. 'Say' puts forward a possibility so that a proposed action can be described which could be used with any set of four boxes. 'You think' on the other hand is used to be more definite about those particular boxes.

However, the use of 'I think' in the responses although also coded as Suppositions does not appear to mark possibilities. Instead, it is used to highlight uncertainty about actual results of actions such as in the following from a Year 8 boy attending a high decile school: 'I think that that one is heavier than that one'.

Being able to describe possibilities is one way of being able to define the conditions under which certain statements are true. This is an important component of being able to describe mathematical generalisations. Although Esty (1992) suggested that this is mainly done through the use of logical connectives, this would seem to be only one way that primary school children do this. Although they can provide information on the conditions under which a statement is true through the use of 'if' at the beginning of an Elaborator in a Premise – Consequence – Elaborator combination, possibilities can also be described through the use of relative clauses as Elaborators in these combinations or by the use of 'say', 'perhaps' or 'probably' as markers of possibilities.

The Weigh Up task was a long, extended task with three related parts. Students' responses to these different parts showed that the structure of mathematical explanations do differ as the task requirements differ. Although the Premise was a text element found in all other tasks, some students, in responding to the Description, did not include a Premise. Instead a new element, Physical Consequence, was more commonly used by students in responding to this part of the task. It would also seem that students who were considered to give accurate and clear responses to this part of the task were most likely to combine Physical Consequences with Consequences. In the other two sections of the task, a Premise – Consequence – Elaborator combination was used by many students who gave clear, accurate responses. This has implications for the types of structures that may support students' being able to provide clear accurate responses, but there is also a need to make students aware of the type of tasks which are most likely to require different text combinations.

Chapter 5

Motorway

Task and acceptable answers

Form of the questions

For this task, the instructions were:

Show student photo (a picture of cars on a motorway) This picture shows a busy motorway. During the day time, about 98 cars go down this road in every minute. 1. About how many cars would go down the road in 9 minutes? **Record student's answer** 2. Explain to me how you got your answer.

Figure 5.1: Question for Motorway task

The form of the questions suggests that a number is expected for the answer to the first question and a description of mathematical manipulation for the second, such as '98 is nearly 100, and I multiplied 9 times 100 to get 900, so it would be a bit less than 900'. An answer that ignored the personal aspect of the explanation would also be acceptable. Such an answer might say 'Nine 100s is 900'.

Answers that were approximately correct

For this analysis, we considered answers between 800 and 1015 to be reasonably accurate, as was a clear statement that the answer was 98 times 9. This level of accuracy is different from that credited by NEMP, which limited the range of credited answers to those between 850 and 900. We chose a somewhat more lax range in analysing language, as it gave us more students whose answers were likely to include mathematical language. The major difference in success on this item, by our criteria, was between Year 4 and Year 8 students.

Of the Year 4 students, 9 gave answers that were within this range. Correct answers came from each group analysed. Three of the six high decile girls gave approximately correct answers, while in other groups one or two students were relatively accurate. Half of the appropriate answers came from the high decile students.

A total of 29 Year 8 students gave answers in this range or gave the calculation as 98 times 9. Again, some correct answers were given by students in each subgroup, between 3 correct answers (low decile non-Pacific boys) and 6 correct answers (high decile girls). There was no overall pattern for success by gender, ethnicity or economic background in the proportion of students who were reasonably correct. See Table 5.1.

Table 5.1. ***Number of students in each subgroup who were approximately accurate on the Motorway task***

		Pacific Low decile	Non-Pacific Low decile	High decile
Year 4	Boys	1	2	2
Year 4	Girls	1	1	3
Year 8	Boys	3	4	5
Year 8	Girls	5	4	6

Form of the answers

The answers that students gave could be classified as numerical, mathematical explanation, non-mathematical stories, and statements that indicated not knowing or uncertainty (e.g. 'I guessed', 'don't know').

All but one student gave a numerical answer. For Year 4 students, these answers ranged between 90 and 1 000 000. For the Year 8 students, the answers ranged between 108 and 48000. Two Year 8 student said that the answer would be ninety-eight times nine and did not estimate what 98 x 9 would equal.

Explanation types of Year 4 and Year 8 Students

There was a difference between groups in the nature of responses given to the request for an explanation, as shown in the following table.

Table 5.2. ***Number of students giving a mathematical explanation.***
(whether or not it was correct)

	Mathematical explanation	Non-mathematical story	No explanation, including guessed, don't know
Year 4 Pacific low decile	1	3	8
Year 4 Non-Pacific low decile	7	3	2
Year 4 High decile	8	1	3
Year 8 Pacific low decile	9	1	2
Year 8 Non-Pacific low decile	10	1	1
Year 8 High decile	12	0	0

Year 8 students gave markedly more mathematical explanations, a finding that is consistent with their greater accuracy. Only one Pacific, low decile, Year 4 student attempted to give a mathematical explanation, while the majority of all other groups at least alluded to mathematical operations. The majority of Year 4 Pacific students did not attempt an answer, saying 'I guessed' or saying 'no' when asked for an explanation.

Non-mathematical stories

Both low and high decile year 4 students gave stories without mathematical content. The nature of these stories is covered more fully in Chapter 7. The stories of Year 4 students included:

> I got no idea. Oh, because they busy? And they waiting for the stop at traffic light. [Q] Because there's many cars, there's heaps. [Q] And they're waiting at the traffic not working, won't work, and that's all (Year 4 Pacific low decile boy)

> Because the motorway's kind of busy and people don't really umm hold up on the motorway, because you can kind of, because there basically isn't a speed limit on the motorway, they go quite fast. (High decile Year 4 girl)

The two Year 8 non-mathematical stories could be considered protoquantitative, using the terms 'too many' and 'lots of' to describe the reason why there would be many cars.

> because there's too many cars. [Q] there's too many cars in the traffic (Year 8 low decile non-Pacific girl)

None of stories told by the Year 4 students had a numerical answer that was approximately right. For the three stories given above, the numerical answers were 89, 52 and a 1000 respectively. The Year 8 student had an approximation of 1000, which could indicate some number sense in the absence of the ability to explain.

In addition to these explanatory stories, some Year 4 students and some interviewers personalised the story. These examples were conversational. For example, one interviewer said 'That's a lot more cars than go by here, isn't it'. A Pacific Year 4 girl asked, 'Is this New Zealand?' Another conversational example came from a high decile boy who said, 'if it was Auckland, it would be a typical day', and then moved to working with decontextualised numbers. This discussion about the picture did not occur with Year 8 students, and may have been an attempt to keep rapport going with the younger students.

For the Year 4 students, these stories suggest that, in the absence of a ready mathematical explanation, a story about the context seemed the best answer to these students. As with the stories told by different economic groups in response to the Bank Account task, the stories were related to their personal experience.

Mathematical explanations

How one does a mathematical operation mentally is not easy to explain, as found in experience with the Numeracy Project (2004). Explanations considered mathematical among those given by these students ranged between protoquantitative ones that did not include numbers and said 'many more' or 'heaps…more', and those that mentioned an operation like 'count them' to those that explained the multiplication followed by repeated subtraction of 2. Examples of these are given in Table 5.3.

Of the nine reasonably correct answers from Year 4 students, four were accompanied by explanations that could be followed. It is possible that the other reasonable answers were guesses that could not be explained or that the student found them in an appropriate manner but then could not explain what they had done. All of the Year 8 accurate answers were easy to follow and usually concise.

Table 5.3. ***Examples of categories of mathematical explanations given by Year 4 students.*** [answer in square brackets]

Protoquantitative or vague	Count, add	inaccurate algorithm	Table	Add and mult	Add, Mult + subtr
Heaps more[200] Yr4 PG	Started repeated addition, can't do it [998]-Yr4HG	9x9=81,8x8=72, put together=[153] Yr4LB	90x10 [900] –Yr4HB	90+90=100smtg, [1000] Yr4HB	doubles/then 980, 992,then repeated subtraction of 2 [882] Yr4HG
8 is back from 9 so you go forward [99]-Yr4 HB	Count them [120] Yr4LG	Hand version of 9 times table [143]-Yr4HB	Times 98 x 9 *done in head without talk* [882] Yr8LnPG	9x9+1x10+1+9 -[909] Yr4HG	98+2=100, 9 100s is 900-2 9 times [888] Yr4 LB
About 1000 – no explanation Yr4	take away some [40] Yr4LG	9 x 98 and then plussed the 2 (sub)answers together [140]- Yr8HB		190+188 until you get 90smtg- [990] Yr4LG,	900, counted all the 2s and minused that –[882]Yr8HB
98 in 1 min, 1/2 would be still coming [1015] Yr4LB	counted on fingers and missed out 2 ones [600] Yr4LG			98+98, take 100 and mult by 8 more [1000]-Yr8 HG	
That's probably roughly what can go down [48000] Yr8LnPB	thought 100, 300, put 98 on that [398] Yr4 LG				
	Plussed 98 with 8 more 98s [872] Yr48PB				
	Because you count it [108] Yr8LPB				

The relatively accurate answers mostly describe a combination of multiplication, addition and subtraction. Given the fact that these students were describing a mental function that was sometimes lengthy, some of these explanations were remarkably good. Even conceptually clear answers included restarted phrases (I did nine, ninety times nine) and a second start at the sentence as below.

> Well, I did nine, ninety nine times nine, I thought well I'm not going into ninety times nine so I just did ninety times ten is nine hundred so. (Year 4 high decile boy)

Note that in this section the boy uses 'did' as the verb to describe his mathematical manipulation rather than 'multiplied', and a colloquial 'I'm not going in to' to describe a choice abandoned.

Another well-described procedure was:

> I just um, went up to 98 plus two is a hundred and nine a hundreds are nine hundred take away.. two nines, oh, um… [Q] Um, I forgot how I done that .. um, it was... [Q] Ninety eight, plus two is a hundred and nine a hundreds are nine hundred take away two, and then, um, take away two nines and I got eight hundred and eighty eight. (Year 4 low decile non-Pacific boy)

The only absolutely accurate answer from a Year 4 student started by describing one method for multiplication, repeated doubling, and then moved to repeated subtraction. This boy thought out loud showing how he found his answer before giving being asked for an explanation. His explanation includes self-corrections and shaking his head when he decided that his way of starting was not going to be useful.

> ... (silent, thinking, then counts very quietly to himself)...which makes a hundred and eighty, six, another one that makes a hundred and eighty .. I mean a hundred and ninety, six, and a hundred and ninety four .. wait (shaking his head)...nine hundred, nine hundred and eighty, nine hundred and ninety two .. eight hundred and ninety .. ninety eight, eight hundred and ninety six, eight hundred and ninety four, eight hundred and ninety two, eight hundred and ninety, eight hundred and eighty eight, eight hundred and eighty six, eight hundred and eighty four, eight hundred and eighty two, eight hundred and eighty two.
> Q
> Um well, what I did is I added up, um, I went from nine, nine hundred from ninety eight and I made that a hundred then I subtracted nine, nine twos from it, from the answer.

In this protocol 'makes' is used for 'equals'.

Other students who gave accurate answers gave considerably less clear explanations, possibly because they were not able to recall how they did the problem. For example, a girl who gave her answer as 998 started using repeated addition in her explanation and then gave up. We are left uncertain about how she got her answer, which could have been done by multiplying 9 x 10 and adding 98.

> Umm .. ninety eight, plus ninety eight, plus ninety eight, plus ninety eight, plus ninety eight, plus ninety eight, plus ninety eight, no, I can't do it. (Year 4 high decile girl)

Another student who gave 1000 as his answer said:

> Because um, ninety and ninety makes a hundred and something .. an then there'll just be ten hundred.' (Year 4 high decile boy)

Incorrect answers sometimes included a recognisable procedure. One student described the way in which nine times tables could be worked out on your fingers or knuckles. Another student described the vertical algorithm, but added the products ignoring the fact that the first amount was not 9 times 9 but 90 times 9.

> …it's fifty, a hundred and um fifty three.
> Q
> ..um, because um, nine times nine is eighty one, and the ninth um eight times nine is seventy two, and I put it together. (Year 4 low decile non-Pacific boy)

The most common answers from Year 8 students were versions of ninety-eight times nine or 100 times 9. These students did not appear to have as much difficulty in explaining the mathematical path to their solution as the Year 4 students did.

Linguistic elements of Year 4 and Year 8 students' answers

This question aroused less discussion than did the other items. As a quick index of the amount of talk from each ethnic and economic group, the number of linguistic elements recorded was totalled. For high decile students, 270 elements were recorded; for low decile non-Pacific students, 184 elements were recorded; and for low decile

Pacific students, 100 elements were recorded. Although rough, this is one index of the willingness of these students to discuss mathematics with the interviewers. It is compatible with the argument that found that students are more likely to carry on conversations with people from similar backgrounds than with people of different backgrounds (Bogdan & Biklen, 1982).

In the following section, totals usually equal more than 100% as students used more than one type of each element in their answers.

Agents

The form of the second question tells the student to: 'Explain to <u>me</u> how <u>you</u> got your answer'. Thus it would be conversationally appropriate for a student to use a personal pronoun in their answer.

Nancy Barron (1971) found that men and women teachers chose to use different grammatical cases in their classroom discussions. She hypothesised that this reflected differences in what was valued by the different gender groups.

> Women produced a significantly greater proportion of explicit participative cases than men, thus demonstrating their greater concern with internal physiological states. The greater involvement of men with implementation of action by means of objects was shown by their greater use of instrumental or source cases. (p. 39)

As a result of this, it seemed interesting to code who or what was the main actor in the majority of clauses of each response. For this task, the actors were either 'I', a generic 'you' or an impersonal actor such as a number or calculation or the question itself. These final type of impersonal actors are discussed further in the section on hesitant language. The distribution of these is given in the following table.

Table 5.4. ***Main actor in responses.***

Actor	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
I	15	14	12	17	8	9	12	29
you	5	4	5	4	3	2	4	9
impersonal	18	18	22	14	16	11	9	36

Certainly the gender difference that Barron had found with teachers was not found with students when responding to this task.

Sixty-one percent of the Year 4 students and 81% of the Year 8 students used 'I', 'you', or an implied personal pronoun as agents in their explanations. Among Year 4 students, Pacific students were more likely to use either 'it' or an existential agent (e.g. 'there are'). This difference is significant (Fisher's exact test 2-tailed p=0.0031). For Year 8 students there was almost no difference amongst groups.

Twenty-two, or 60% of Year 8 students used numbers as the agent of their sentences or clauses, as in 'because 98 plus 2…', while only six (17% of the Year 4 students) limited their explanations to having the numbers as the agent. For both year levels, this use was spread across the groups. The difference appeared to be related to competence in the Year 8 students' ability to strip away the story shell and deal with the mathematics only, while the younger students talked about what they did or what the cars did.

Processes

This was the one task of the four considered in this study for which mathematical verbs predominated for both year levels. These verbs included 'times', 'take away', 'equals' and 'estimate'. One third (33%) of all verbs by Year 4 students and nearly half (48%) of those used by Year 8 students used were considered mathematical. The second largest category was static verbs.

In Year 4, this preponderance of mathematical verbs over all other types of verbs is mostly due to the upper decile students who said more and gave more mathematical explanations (Fisher's 2-tailed test $p = 0.0472$).

In Year 8, the highest proportion of mathematical verbs was used by low decile non-Pacific boys but this proportion was influenced by a high number used by two of the boys who gave extensive answers. See Table 5.5.

Table 5.5. ***Types of action terms (verbs) used by each main category*** *(combining girls and boys).*

	Total action terms (verbs)	Mathematical	Static	Non-mathematical action	Other
Yr 4 High decile	79	37	29	7	6
Yr 4 Low non-Pacific	57	17	13	15	12
Yr 4 Low Pacific	30	0	12	14	4
Yr 8 High decile	70	32	61	4	7
Yr 8 Low non-Pacific	73	31	28	7	8
Yr 8 Low Pacific	25	18	6	0	1

Abstractions and names of numbers

As might be expected in a problem where numbers were given, the names of numbers were used much more frequently than abstractions like 'all' or 'every'. This was true for both economic groups, both ethnic groups and both genders in Year 4, but was only used by one student amongst all those in Year 8.

Comparison

Expressions of comparison were made only by those Year 4 students who did not work out the mathematics of the problem, and by none of the Year 8 students. An example was '… if there's so many go down in one minute will there be so many more have to go down in nine minutes' (Year 4 high decile girl).

Connectives

In response to this question, Year 8 students used more causal connectives ('because', 'so') than additive ones (primarily 'and'). The overall numbers were small, with 31 causal connectives and 27 additive connectives being used by the Year 4 students. The Year 8 students used a total of 43 causal connectives, most of which were 'because' and 'so', and 51 additive connectives, 36 of which were 'and'. Where 'and' was used repeatedly, this was usually because the multiplication problem was done as repeated addition. Causal connectives are appropriate for mathematical explanations. An example of this use was, 'you'd use a times so you would do ninety times, ninety times, ninety times nine…´(Year 4 high decile boy).

Text structure

Motorway, like Weigh Up, requires students to explain what they did, in this case to get the number of cars which go down the Motorway. Table 5.6 shows which students used clear language in giving their explanation. LPG stands for Pacific girl attending a low decile school. LnPB stands for a non-Pacific student who attends a low decile school, whereas HG stands for girls attending high decile schools. The spread is much more complicated that that for Better Buy. This is not just because the students were using a range of different strategies for solving the problem, but there was also far greater spread of answers which were categorised as clear, moderately clear, vague or elliptical. Once again, it was not always those students who had an accurate answer or close estimate who had the clear language. Bills (2002) described work by Piaget (1928), who felt that having been asked 'how they had performed a calculation, after giving an answer by some automatic process, children may invent something which would give rise to the same answer' (p. 99). It may well be that, as some students' explanations did not always seem to match the amounts they gave, that similar inventions of explanation also occurred in this task.

Table 5.6. ***Clarity of language vs. accuracy for explanation**.**

	Clear language	Moderately clear but some specific information missing	Unclear, vague	Elliptical
Uses rounding to provide an accurate answer	1Yr 8 LnPB 1 Yr 4 LnPB	1 Yr 8 LPG		
Uses rounding to achieve an estimate	1 Yr 8 LPG 1 Yr 8 LnPG 2 Yr 8 HG 1 Yr 8 HB 1Yr 4 HB 1 Yr 4 HG	3 Yr 8 HG 1 Yr 8 HB 1 Yr 4 LnPG	1 Yr 8 LPB 1 Yr 8 HB	1 Yr 8 LnPG
9 x 98	2 Yr 8LPG 2 Yr 8 PLB 3 Yr 8 LnPG 2 Yr 8 HB 1Yr 4 LnPB	1 Yr 8 LnPB 1 Yr 8 LnPG 1 Yr 4 HG		1 Yr 8 HG
Other mathematical calculations	1 Yr 8 PLB 1 Yr 8 LnPG	2 Yr 8 LnPBs 1 Yr 4 HG	1 Yr 8 LnPB 1 Yr 8 HB 1 Yr 4 LnPB 1 Yr 4 LnPG	1 Yr 4 HB 1 Yr 4 LnPG
Doesn't use information mathematically	1 Yr 8LPG 1 Yr 8 LPB 1 Yr 8 LnPB 1 Yr 4 LnPB 1 Yr 4 HB	1 Yr 4 LnPG 1 Yr 4 HB	6 Yr 4 LPGs 4 Yr 4 LPB 1 Yr 4 LnPG 2 Yr 4 LnPBs 3 Yr 4 HG 1 Yr 4 HB	1 Yr 8 LPB 1 Yr 4 LPG 1 Yr 4 LPB 1 Yr 4 LnPG 1 Yr 4 HB

*Student identification as in Table 3.1

There were 18 responses which used estimations to gain either an approximate or exact amount. Over half of these were by students attending high decile schools, twice as many students as those from low decile schools, either Pacific Islanders or not. There were equivalent numbers of boys and girls, but more than three quarters were students in Year 8. Similar results for year level and gender can be seen for the 14 responses which gave '9 x 98' for their calculation. However, there were no significant differences in the decile level of schools attended. At the other end of the table, of the 20 students who were unable to use information mathematically, 7 students were from high decile schools. Six students were from low decile schools in Pacific communities. There was a similar even distribution by gender. One difference in distribution was that 16 of these 20 students were in Year 4. This suggests that students from high decile schools were most likely to use estimates whilst non-mathematical responses were more evenly spread across groups.

Of 26 students who used clear language, 20 were in Year 8, suggesting that clear language is related to age regardless of the mathematical ability of the student. Students were clearer in Year 8 whether they gave an appropriate mathematical response or not. There were no other differences according to gender, decile level of school attended or ethnicity. There were only 8 students who gave elliptical responses with no clear distinctions between any group.

The Motorway task required students to provide an explanation of their calculation to determine the number of cars. As such, it shared many similarities with the Weigh Up task which also required an explanation in its Plan and Explanation sections. It was, therefore, not surprising to find many of the same text elements being used; that is Premise, Consequences, Conclusions and Elaborators. The distribution of students using these elements is given in Table 5.7.

Table 5.7. ***Use of text elements by different groups.***

Text Elements	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
introduction	5	6	4	7	1	6	4	11
premise	36	36	36	36	24	24	24	72
consequence	15	14	13	16	3	10	16	29
conclusion	7	10	8	9	0	7	10	17

Apart from Introductions which appear to be used more by Year 8 students (and the numbers are not large), there do not appear to be differences in distribution of students based on gender or age for any of the text elements. However, there are significant differences due to ethnicity. On the whole, Pacific students did not use any text elements apart from Premises. Non-Pacific students who attended a similar decile level of school used slightly less Consequences and Conclusions than students attending high decile schools. Interestingly, it would seem that these students used more Introductions than those from high decile schools. However, the numbers are small and no clear trends can be determined. These differences between Pacific students and others can be seen clearly in Table 5.8, which shows which students used particular text combinations.

Table 5.8. ***Use of combinations of text elements by different groups.***

Text Structures	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
intro – premise	3	0	0	3	1	2	0	3
intro – premise – consequence (+ conclusion)	2	6	4	4	0	4	4	8
premise	17	18	20	15	20	10	5	35
premise – consequence	8	7	7	8	3	4	8	15
premise – consequence – conclusion	5	1	2	4	0	2	4	6
premise – conclusion	1	4	3	2	0	2	3	5

More students attending high decile schools used combinations of three or more text elements. As was seen in Table 5.6, Pacific students predominantly only used a Premise. The next few tables go through each of the text elements and provide more details before looking at what combinations of text elements were most likely to be related to clear language and accurate responses.

Introductions only occurred at the beginning of the response and seemed to personalise it. For example, one boy stated

> Um, I go for, I go to nine times nine then eight times nine, and then, then I, then I plussed the two answers together and got around, that.

'I go to' is setting the scene in that he is describing what he did before going on to describe the actual process.

As seen in Table 5.9, eleven students began their responses with an Introduction. All were followed by a Premise and two continued with a Consequence, while another six used a Premise, a Consequence and a Conclusion. It was, therefore, more likely that an Introduction was found in texts which incorporated a longer, more complex series of elements.

Table 5.9. ***Use of Introductions by different groups.***

Text Structures	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
introduction - premise – consequence – conclusion	1	5	3	3	0	3	3	6
introduction - premise – consequence	1	1	1	1	0	1	1	2
introduction - premise	3	0	0	3	1	2	0	3

These numbers are small with no clear trends based on gender or decile level of school attended. It would seem that Year 8 students were more likely to employ an Introduction than Year 4 and students from high decile schools were more likely to use them than students from Pacific communities, in low-decile schools. The distribution of groups using an Introduction are similar to those without an Introduction as can be seen in Table 5.8.

Table 5.10. ***Use of Conclusions by different groups*** *(including those with Introductions).*

Conclusions	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
Conclusion preceded by consequence	6	6	5	7	0	5	7	12
Conclusion preceded by premise	1	4	3	2	0	2	3	5

In Table 5.10 there are no clear distinctions in who used Conclusions on the basis of age, although there does seem to be a slight tendency for boys rather than girls to use them. No Conclusions were found at the beginning of text structures, unlike what had occurred in the Better Buy responses. It was far more likely that students from high decile schools included them in their responses. Unlike the Better Buy texts, only two of these Conclusions were implicit. One other small difference is that Year 8 students were more likely to have a Conclusion preceded by a Consequence, whereas a Conclusion preceded only by a Premise was more likely to be used by a Year 4 student. However, the numbers are small and any pattern remains uncertain.

Table 5.11 shows the distribution of students who concluded their responses with a Consequence. As there were differences between Premises, the type of preceding Premise is also provided. Premises are divided into types based on the kind of evidence they used. A factual Premise used information given in the question or from the photo which was supplied, for example 'because it says nine minutes', 'because there's lots of cars going up and down'. A mathematical Premise used a mathematical calculation or made reference to a mathematical fact, such as 'I rounded ninety-eight up to, umm, one hundred' or 'ninety and ninety makes a hundred and something'. When students used more than one Premise, they were categorised under the first Premise or the type of Premise used most often.

Table 5.11. ***Use of Consequences as the final element and their preceding Premise*** *(includes responses with an Introduction).*

Consequences	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
consequence preceded by factual premise	3	1	2	2	2	0	2	4
consequence preceded by mathematical premise	6	7	6	7	1	5	7	13

Once again, the numbers are small with few distinguishing differences between groups. Similar numbers of Year 4 and Year 8 students used this structure. However, there were more students using a mathematical Premise than a factual Premise and these extra students were more likely to be boys, who were in Year 8 and from a high-decile school. It would also seem that Pacific students attending low-decile schools were least likely to use text structures which ended with a Consequence. This is in contrast to students who only provided a Premise. These results are shown in Table 5.12. When these results are compared with those of students who used a Conclusion, it can be seen quite clearly that students from high decile schools were more likely to provide their responses in complex text structures.

Table 5.12. ***Use of only Premises in the text****.*

Premises	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
factual premise	6	3	8	1	5	4	0	9
personal premise	3	8	10	1	7	2	2	11
mathematical premise	8	7	2	13	8	4	3	15

Of the 24 Pacific students, twenty used only Premises. Of these, twelve were either factual, repeating information in the question, or personal. Personal Premises were related to an action that the speaker did which was not mathematical, for example 'I don't know I just guessed'. Personal Premises, because of their nature, did not occur with a Consequence and/or Conclusion. It would seem that more Pacific students were likely to feel that admitting a lack of knowledge was an acceptable way to respond to a mathematical assessment task. However, it was also the case that 8 Pacific students recognised that mathematical calculation was required and were able to provide an appropriate one. For example a Year 8 girl said, 'Nine minutes times ninety-eight cars'. Yet, knowing this did not mean that they could actually provide an appropriate numerical amount, as many found doing this calculation too hard to do mentally and had no other way of arriving at an approximation. 4 Pacific Year 8 students did the calculation and gave an accurate response by just supplying a Premise. However, if only a mathematical Premise was given then it was unlikely that the student would be able to solve the problem appropriately. The number of Pacific students who only used a Premise was double the number of students from low decile schools in areas with a low Pacific population.

In considering how the different elements were connected, there were not the number of Premises beginning with 'if … ' as there had been in the Better Buy task. This can be seen in Table 5.13.

Table 5.13. ***Use of logical connectives in the text structure.***

Logical Connectives	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
'if' before premise	5	3	7	1	2	2	4	8/72
'so' before a consequence	4	6	5	5	0	5	5	10/29
'so' before a conclusion	2	1	2	1	0	1	2	3/17
'and then' before consequence	1	7	2	6	0	5	3	8/29
'and then' before conclusion	0	1	0	1	0	0	1	1/17
'and' before conclusion	1	5	3	3	0	3	3	6/17

When looking at this table it is worth remembering that from the total of 72 students, only half used a text structure which included more elements than just the Premise. The total numbers of students who used each of the elements is given as the second number in the totals column.

The difference in use of logical connectives by gender is interesting. 18 boys compared to 19 girls used text structures which consisted of more than just a Premise element. Yet, on the whole, boys used more logical connectives than girls.

In regard to age, unlike the Better Buy task, it would seem that Year 4 students were more likely to place 'if' before a Premise. Five more Year 8 than Year 4 students had used more than just a Premise in their responses, but it would seem that proportionally there is no significant difference in the use of logical connectives according to age.

A similar situation happens when decile level of schools is considered. As so few Pacific students used more than a Premise in their response, it is not surprising that they used few logical connectives. However, it was expected that as students who attended high decile school used more complex text structures, that they would also use more logical connectives than non-Pacific students attending low decile schools. However, this was not the case, with both groups using almost equivalent amounts of logical connectives.

Within the text structures, there were also clauses which provided more information to particular elements. For example within a mathematical Premise such as '98 is almost a hundred, so it's just two away', 'so it's just two away' adds more information about 98 being almost a hundred. This extra information is a known fact rather than as a consequence of working with 98. Therefore, it cannot be considered as a second Premise, as it does not add new information. Nor can it be considered as a Consequence, as nothing was done to 98 for a new idea to be developed as a Consequence. Consequently, these types of clauses have been labelled as Elaborators. Other Elaborators were clauses following what Halliday (1986) calls mental processes such as 'say', 'think' or other verbs that were followed by 'that' or 'which' clauses. These following clauses have also been labelled as Elaborators. For example, one child responded with 'I counted all the twos that were needed to go all the way up to nine hundred'. 'That were needed' and 'to go all the way up to nine hundred' both provide an elaboration of the previous clause by providing more information about it. These are considered cohesive devices (Hudson & Shapiro, 1991). Where these Elaborators occurred is set out in Table 5.14.

Table 5.14. ***Use of Elaborators in the text structure.***

Elaborators	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
Within a introduction	1	1	2	0	0	0	2	2
Within a premise	4	3	3	4	2	2	3	7
Within a consequence	1	3	1	3	0	1	3	4
Within a conclusion	1	1	1	1	0	1	1	2

The number of Elaborators used in responses to this task were quite small and there are difficulties with discussing any trends. Given that every response included a Premise, it is not surprising to find that Elaborators tend to appear more often with Premises. However, it would seem that they were more likely to be used by students from high decile schools than by other students.

Clear language and correct answers

Given that what counted as an accurate answer for this task was much more diverse than for the Better Buy task, it was expected that the linguistic structures used might also be more diverse. Table 5.15 gives the text structures for the 15 students whose responses were described as clear and who had a good estimate (within 100 of the exact answer) or an exact answer.

Table 5.15. ***Text structures used by students with clear language and who gave good estimates or exact answers.***

Text Structures	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
intro – premise (mathematical) – consequence – conclusion	0	1	1	0	0	1	0	1
intro- premise (mathematical) – consequence	1	0	1	0	0	0	1	1
premise (mathematical)	3	3	0	6	4	1	1	6
premise (mathematical) – consequence	2	2	0	4	1	1	2	4
premise (mathematical) – consequence – conclusion	2	1	1	2	0	1	2	3

Of the fifteen students who used clear language and also provided a good estimate or exact answer, six of them only used Premises and four used a Premise, Consequence and a Conclusion structure. This would tend to suggest that for this task there was not a high correlation between getting a correct answer and using a more complex text structure containing a series of different elements. However, the use of mathematical Premises did seem to be important. It is worth noting that Pacific students who were able to get a correct answer were all students who only provided mathematical Premises within their answers.

There were a further 15 students out of the sample of 72 who were able to respond to this task correctly. These students gave a moderately clear or vague response. As can be seen in Table 5.16, Year 8 students attending high decile schools were the most likely groups to give these responses, but their combinations of text elements were much more varied than those given with clear language. For example, some students used a factual rather than a mathematical Premise as the basis for their explanation. In giving clear explanations, only 6 students did not have a Consequence as one of the elements in their response. However, out of the 15 students who gave an unclear response, 6 students also did not include a Consequence but only 3 just gave a Premise.

Table 5.16. ***Use of text structures by students with unclear language and who gave good estimates or exact answers.***

Text Structures	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
intro – premise (mathematical)	1	1	0	2	1	1	0	2
premise (mathematical)	2	1	0	3	2	0	1	3
premise (mathematical) – consequence	0	4	1	3	0	2	2	4
premise (factual) – consequence	0	1	0	1	0	0	1	1
premise (factual) – conclusion	1	0	0	1	0	0	1	1
intro -premise (mathematical) – consequence – conclusion	1	1	0	2	0	1	1	2
premise (mathematical) – consequence – conclusion	1	0	0	1	0	0	1	1
premise (factual) – consequence – conclusion	1	0	0	1	0	0	1	1

The text structures used students who gave incorrect answers generally only contained Premises. At Year 8, many students, especially from low decile schools were unable to give an estimate, but did give the mathematical Premise '9 times 98' or the equivalent. Year 4 students were more likely to give a factual Premise such as 'Heaps a go, heaps a cars go down the road every day and in nine minutes … more cars come in' or a personal Premise such as 'I don't know I just guessed.'

In the responses to the previous tasks, students who gave accurate answers were more likely to use a complex text structure involving the combination of several text elements. In the Motorway responses, it would also seem that it was mostly Year 8 students who could do the mathematics who would use more complex text structures. What Motorway also illustrates is that just providing a Premise can sometimes be considered a clear response. However, on the whole students, who just gave a Premise, who were predominantly Pacific students were most likely to be unable to provide an accurate response.

Hesitant language

It was apparent that many students were hesitant in giving their responses, either because they were uncertain about their own mathematics or because they felt that the situation required them to show deference to the teacher administrator. In the Weigh Up task, showing hesitancy is discussed with the use of Suppositions. In the response to this task, there appeared to be two different ways that students made their responses seem hesitant or vague. The first way was in regard to expressing uncertainty about the correctness of their thinking. Making a response vague can decrease a student's risk of losing face. The second relates to the 'estimation' element of the task and illustrates how students mark the approximation of their calculations and answers.

By using hesitant language to limit a potential loss of face, students are exhibiting signs of what Grob, Meyers and Shuh (1997) labelled *powerless language*. 'Powerless language typically has been defined as speech marked by hesitancy and tentativeness … it often contains more polite forms, hedges, hesitations, disclaimers, intensifiers, empty adjectives, tag questions, and hypercorrect grammar' (p. 284). In the responses given by students to this task there were examples of hesitations through verbal fillers, hedges and disclaimers. The distribution of these is given in Tables 5.17, 5.18, 5.19. There were also two uses of intensifiers. One Year 8 boy from a high decile school recruited support for his answer by including the word 'obviously' in his response.

> Well, each minute ninety-eight cars come down, says about, so obviously that's about nine hundred, says nine minutes and then one minute goes ninety-eight cars.

A Year 4 girl used 'basically' in a similar way, although it did not have the strength of 'obviously'.

> Um.. because the motorway's kind of busy and people don't really um hold up on the motorway, because you can kind of, because there basically isn't a speed limit on the motorway, they go quite fast.

Burton and Morgan (2000), in their study of the professional writing of mathematicians, found intensifiers were used quite frequently as authority claims. It is interesting to see that students in responding to this task do not claim authority for the correctness of what they have done in the same way. The same student who used 'obviously' was also one of only two students who referred to the question as support for what they had done. The other student was also a Year 8 boy from a high decile school.

> Um, because it says nine minutes so I just went up to nine hundred.

The 'it' has become the authority on which the student is basing his argument. This would be what Sowder and Harel (1998) had described as an external based proof as it calls on an outside authority to authenticate the evidence.

Other negative politeness strategies which make the response seem vague were when the speaker added a disclaimer. Disclaimers are expressions such as 'I mean' and 'I think'. These suggest the speakers' uncertainty about the statement which follows, whereas intensifiers tend to recruit the hearer's support for the truth of the statement. Table 5.17 sets out the distribution of disclaimers in the responses to this task. The small numbers of students who use disclaimers make it unclear whether there are any patterns associated with disclaimers, except that five out of seven students were from high decile schools. When the information on disclaimers is combined with intensifiers, it would appear that these are more likely to be used by students from high decile schools. If these linguistics features are markers of powerless language, it is surprising to find that they are more likely to be used by these students, who were the most likely in Table 5.17 to be considered to have clear language.

Table 5.17. ***Use of disclaimers in the text structure.***

Disclaimers	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
I mean	1	1	1	1	0	1	1	2
I think/I thought	3	1	3	1	0	1	3	4
I said	1	0	0	1	0	0	1	1

There were quite a number of 'verbal fillers', as described by Brown (1977), in the introductions. These words mark a request to take a conversational turn but give the speaker time to arrange their thoughts without leaving a silent period that could be filled by another speaker. Rowland (2000, p. 136) labelled these as 'maxim hedges' as by using them 'the speaker is serving notice to the hearer that the contribution about to come will in some respect fall short of one or more of Grice's maxims'. These maxims relate to the pragmatics of the text which deal with ensuring that the communication between hearer and speaker is as clear as possible. Four responses began with 'umm' and another used 'umm' as the third word. Two continued with 'well' and another two explanations began with 'well'. The distribution of fillers at any point in the responses is given in Table 5.18.

Table 5.18. ***Use of fillers by different groups.***

Fillers	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
umm	16	15	17	14	6	9	16	31
oh/ah	4	8	3	9	5	5	2	12
well/okay	4	6	5	5	0	2	8	10

On the whole, it would seem that there are no large differences in distribution of fillers according to gender. There is a small difference according to gender with slightly more boys using them than girls and according to age with Year 8 students using them slightly more often than Year 4 students. Pacific students were the least likely to use fillers whilst students attending high decile schools were the most likely, especially in regard to use of 'well/okay'. Given that these were the students most likely to have an accurate response, it is surprising to find that they more likely to use these features of hesitant language.

In the Introductions, four of the responses contained 'just' after 'I', such as 'I just went' and 'I just took'. In the rest of the explanations, there was also 17 other students who used 'just'. 'Just' acts as what Rowland (2000. p. 60) described as an *approximator* which modifies a proposition by 'making it more vague'. The need for such vagueness is, perhaps, more to do with uncertainty about the correctness of their approach rather than uncertainty about what they remember that they did. As the question specifically asks students to 'explain to me how you got your answer', these Introductions could be responding to this personal flavour of the question. Given the power relationship which exists between the student and the teacher administrator, it is not surprising that the student may be more hesitant to talk about how they got their answer than to simply give the numerical result. Brown and Levison (1978) suggested that when the Hearer has high power over the Speaker, the Speaker will tend to use negative politeness strategies, such as those which lessen the strength of the utterance and to use off-the-record strategies such as putting forward ambiguous statements. As well as adding 'just', students also used words such as 'kind of', 'maybe' in their responses to make their responses appear less certain. In mathematics classrooms, Rowland (2000) suggested that students are often vulnerable when responding to teachers because of the fear of ridicule. By suggesting through negative politeness that they are unsure of their thinking process, students reduce the loss of face if their thinking is not appropriate. Table 5.19 provides a description of variety and distribution of hedges used in the responses to this task.

Table 5.19. ***Use of hedges by different groups.***

Hedges	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
just	6	14	8	12	4	9	7	20
like	2	5	2	5	2	3	2	7
kind of/sort of	1	1	2	0	1	0	1	2
maybe	2	2	3	1	2	1	1	4

Hedges were used more often by boys, especially 'just', but there was little difference according to age or to decile level of school attended. There seems to be slightly less chance that Pacific students would use hedges than other students.

In considering who uses what examples of hesitant language, it would seem that boys are more likely to use hedges. Year 8 students are more likely to use fillers and Pacific students less likely to use fillers or hedges. This last result could be because these students gave the shortest responses. However, the relationship between short responses and the need for fillers or hedges is not clear.

It has been suggested that 'powerless' language contains more intensifiers, hedges, disclaimers and hesitations and is generally associated with female language (Grob, Meyers & Shuh, 1997). Yet the tables in this section clearly show that boys use more hedges and hesitations than girls do in presenting their responses to this task. This appears to match other research done in this area. Staley (1982) found that for children talking informally in groups, boys aged 4, 8 and 16 used more hedges. It was only with twelve year olds that it was girls who used more hedges. 16 year old boys used significantly more hedges. In work undertaken by Grob et al (1997), female college students used more hedges than males, but the amounts were not significantly different. Even if hedges are considered to show politeness rather than powerlessness (Fishman, 1978), studies by researchers such as Deucher (1990) suggests that women, including girls, generally use politer language than men It is quite surprising then to find that in describing a response to a mathematics task, boys who typically are believed to feel more confident with their mathematics ability (Burton, 2001) expressed themselves in a more hesitant manner.

The other way that students express hesitant ideas is when they provide approximate calculations or answers for the task. Given that the task asked for an approximate answer ('About how many cars would go down the road in 9 minutes?'), it is perhaps more surprising that very few student did this. There are two types of approximators:

rounders and adaptors. Rounders are words such as ‘about’, ‘around’ and ‘approximately’, whereas adaptors are words and phrases such as ‘a little bit’ and ‘like’. In giving their answer, students used expressions such as ‘really, that’s probably about roughly what goes down’ and ‘probably get around a thousand’. Table 5.20 provides the distribution of students who used the rounder ‘about’ or an adaptor such as ‘like’ or a rising intonation to indicate uncertainty about the amounts given in response to the first question of this task.

Table 5.20. ***Use of approximators in answers to the first question.***

Approximators	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
about	5	9	5	9	1	5	8	14
Other approximators	4	5	4	5	0	1	8	9
Rising intonation	3	0	1	1	1	1	0	2

The numbers are small, with the only discernable trend being that students from high decile schools are more likely to use rounders or adaptors than students from low decile schools, particularly Pacific students. It may also be that Year 8 students were more likely to use a rounder than Year 4 students. However, these results are not as clear cut as those reported by Rowland (2000), which had shown that students could determine when it was appropriate to use a rounder by the age of eight, which would be the age of most students in Year 4.

Students also used approximations in their explanations of how they had determined their answers in discussing their calculations. In discussing amounts, students used expressions such as ‘like, really big’, ‘only three’, ‘heaps of cars’.

Table 5.21. ***Use of approximators in giving their explanations by different groups.***

Approximators	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
For discussing amounts	6	4	6	4	2	5	3	10

The results for Table 5.21 support those in Table 5.20 in that there appears to be no difference by gender or age in using approximators. However, unlike the results in Table 5.20, there does not seem to be a differences due to decile level of school attended. This may be related to the very small numbers of students who used approximators in this position.

Questions

Many students asked questions. Most of these were requesting or clarifying information given in the task problem, for example, ‘is this like an estimate, kind of?’ or ‘ninety cars goes down in one minute, eh?’ Many of them featured hesitant language such as ‘kind of’ or tag questions such as ‘eh?’ These examples of hesitant language were not included in the tables above, as they were asked in response to the first question of the problem rather than during their explanations of their calculations. It is to be expected that students would exhibit politeness strategies when requesting the teacher administrator to repeat already given information or to clarify what they had to do. These requests have the potential for the speaker to lose face, as they are admitting that they did not listen accurately or they are uncertain of what they have to do even if they did hear the problem completely. Two students also asked self-questions and four students turned their answers to the first question into questions either by using a question format or finishing their statements with a rising intonation. Table 5.22 shows the distribution of questions.

Table 5.22. ***Use of questions by different groups.***

Questions	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
Of self	0	2	2	0	0	0	2	2
Of the teacher administrator	11	11	9	13	8	7	7	22
About their answer	2	2	2	2	1	2	1	4

There appears to be no obvious patterns in who asked questions. Slightly more Year 8 students asked questions of the teacher administrator than Year 4, but the difference is not great.

It would seem that in responding to this task, students used a range of different strategies to make their responses seem more vague. Although there were a few differences in the distribution of groups using them, there appears to be no clear pattern. For example, Year 8 students from high decile schools were most likely to use approximators, but Pacific students were least likely to use hedges. As an area which is likely to influence teachers' perceptions of students' ability, this area of mathematical language use requires more investigation.

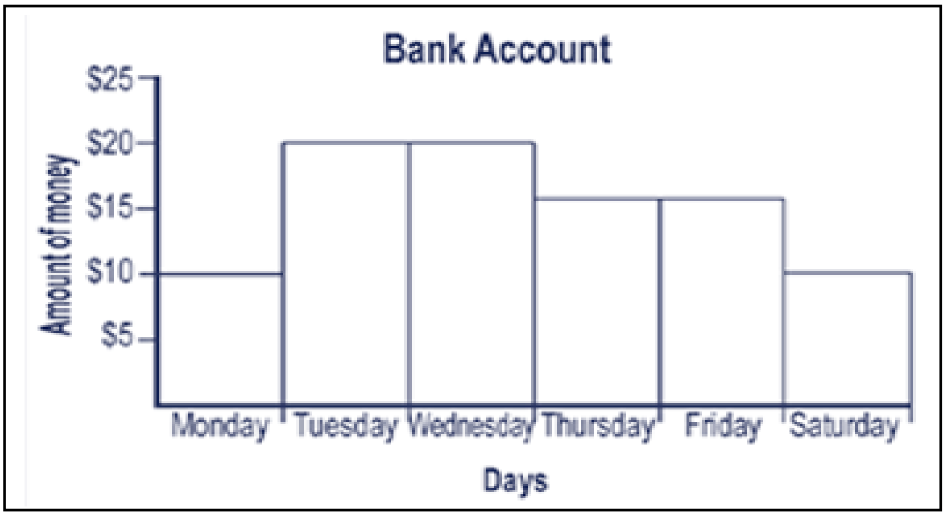
Conclusion

The Motorway task required students to explain how they arrived at the number of cars going down a busy Motorway. Students' responses varied from those who refused to respond verbally to those who gave accurate amounts by using good approximation strategies. Although there were differences in not only who gave appropriate responses and who used clear language, there was no clear relationship to the text structures used in their responses. There also appear to be no clear patterns in who used hesitant language. Previous research had suggested that girls were more likely to use more hesitant language to exhibit powerlessness or politeness but if anything, this research suggests that boys used more hesitant language. It was also expected that those with clear language were more likely to use non-hesitant language, but this was not the case. The responses for this task are, therefore, intriguing and further research is needed.

Chapter 6

Bank Account

For this task, students were shown a graph of the amount of money in a bank account on six consecutive days. The instructions for what the teacher administrator was to say and do for this problem were as follows. Instructions of what to do are given in bold.



Put graph and ruler in front of the student
This graph shows someone's bank account
Point to the amount of money
Up this side is the amount of money the person has.
Point to the days
Along the bottom are the days of the week
Have a careful look at the graph and then tell me a **story** to explain what is happening with the money (bold in the original).
Point to the beginning of the graph

Figure 6.1 Question for the Bank Account task

The final request to 'tell me a story…' asks for a personal response and for a story. The statement, 'Tell me a story to explain what is happening with the money' was not heard by some students as a request to explain deposits and withdrawals, but merely the amount of money in the account each day. There was no request for the mathematics to be foremost, or for mathematical language to be used. This disguised request for an explanation of a mathematical representation complicates the need to display understanding of the mathematics presented in the graph as students are expected to cross between modalities, from the modality of mathematics to that of providing a story that matches the graph.

A story that included reference to putting money in and taking money out so that there was more or less the following day as shown by the next bar would include the expected elements. However, conversational stories rarely include all of these elements. They are often elliptical, expecting the listener to understand that if you took money out of course the graph would show a lower level the next day. In formal mathematical expression, the causal connectives are essential.

In this chapter we first discuss the accuracy, type of stories told, and linguistic features by age group. Then we present a section on the explanatory text structures used. This is followed by an analysis of the hesitancies in the students' language.

Year 4 students, story type

Some of the Year 4 students took the request to tell a story literally and told elaborate stories, although these were not always stories of increases and decreases in a bank account. It appeared that when they gave only a description of the graph, the interviewers prompted them further for a story. Few stories reflected experience with a bank balance that changed daily. Stories were of several types, some stories being of more than one type. Some of these stories could be considered to be reflective of the students' backgrounds or their interest in stories from books or the news. Several associated the graph with only deposits or only withdrawals. In one case, this was a story of money being raised. This may have come from experience with school or church fund-raising where the amount raised daily had been displayed. These stories could be appropriate had the table been labelled 'Amount deposited in (deducted from) a bank account', rather than the amount the person 'has'. The appropriateness or inappropriateness of an answer depends on understanding of one word from the interviewer. This may not be enough of a clue to override the students' personal experience. These stories are discussed further in Chapter 7.

Table 6.1 gives the type of story told by each group of students. By the criteria expected for the test, only the first column would be considered correct. The totals for each group may be greater than six if one type of story was given and then they were pressed by the teacher administrator.

Table 6.1. ***Focus of stories told by Year 4 students for the Bank Account task.***

	Both save & spend or lost	Amount had or description of graph only	Add only	Deduct only	Story without reference to changes on graph	No explanation
Yr 4 Pacific low decile boys	1	1		2	2	
Yr 4 Pacific low decile girls		1	2	2	1	
Yr 4 non-Pacific low decile boys	2	3				1
Yr 4 non-Pacific low decile girls		3	1	2		
Yr 4 high decile boys	2	4				
Yr 4 high decile girls	2	4				
Total	7	16	3	6	3	1

Thus 13 of the 36 students did not give the type of story that was expected. All of these were lower decile students. Nine of these were Pacific and four were non-Pacific students. These differences between economic groups were highly significant (p=0.00029)

Seven students did as the question expected and made up a story to explain the change of the graph. Sixteen students simply reported what had happened on the graph with no causal statement. This response was most common among the high decile students and least common among the Pacific, low decile students. This suggests that students who were mostly of the same ethnic group as the interviewers believed that the mathematics of graph reading was what was important, not the story.

There were marked differences in the type of stories told by these Year 4 students. Three of 12 Year 4 Pasifika boys from low decile schools mentioned robbers or stealing in their stories as did one Year 4, high decile girl. Stories of lower decile children included 'they borrowed it because they had no money, they needed it to buy food, to buy lunch for their daughter, for school' (Pasifika girl); 'mum hates going to the bank' (Pasifika boy), 'when my dad gets paid, yeah, he'll put money in the bank for me'(Pasifika boy) and money owned by 'the people' (non-Pasifika boy, Pasifika boy).

Three Year 4 boys from high decile schools talked of the money going up and down, possibly treating it like the share market or possibly just referring to the graph, thus ignoring the story shell expected by the final question. They commented that the graph went up or down but did not attribute an external agent to these changes. An example of this was:

> Well, the money, on Monday it is, it was on ten dollars, then on Tuesday it grew (unintel) it went to twenty dollars, and on Wednesday it's still on twenty dollars, then Thursday it lowered down, and it was (pause) on fifteen dollars, then on Friday it was fifteen dollars again and on Saturday it was on ten dollars.

Two high decile girls treated it like an exercise in story telling. One started her story 'Once there was a little boy named James and he had lots of money…'. The other included specific purchases made, as in 'he went to the store and bought some sugar and some cherries'. These story telling skills are similar to those encouraged in school, where emphasis is placed on writing and editing good stories as an aspect of literacy. For these girls, the story telling aspect seems to have been more important than the mathematics in the representation.

Year 4 Linguistic features

Agents

The agents in the stories told were other people, not the student. One girl from a high decile school asked 'how do I know if it is a man or a girl?' This was one indication that most students did not see bank accounts as something that they owned or looked at regularly. When 'I' was used it was usually in conversation, as in 'I don't know', or 'I mean'. Within the explanatory story, only one child used 'I' as the agent of her story, and this was a modification of a clause that started 'Monday we had, I had ten dollars'. Agents were usually the same throughout a story, although a story might use 'the people' initially and then 'they' for later references to the person. Counting only the main agent in a story, 'they' was most common (8) followed by 'he' (4). Other agents used were: the person: 2; the people: 2; someone: 3; robber 2; my dad, the man, and she.

For the students who reported on the characteristics of the graph, bars or amounts of money were referred to as 'it'. This occurred in 11 cases, sometimes in conjunction with the personal agents listed above.

Actions

Many of the verbs used were specific to money (19%). These included 'put in', 'stole', 'saved', 'spent', 'borrowed', 'costed' [sic], 'earns' and 'gets paid'. All of these would be specific to the context and inappropriate for an identical graph that was, for example, of rainfall. They indicate that students were attending to the story context that was given to them. Some of these verbs, like 'spent' were consistent with the mathematical comparisons required while others were not.

More common were verbs that referred directly to the comparison of the height of columns (26%). Some of these were not strictly comparative. These included 'grew', 'went up'. Others might be considered transitional to mathematical expressions, like 'goes bigger' and 'goes smaller'. Others, all used by one student, refer more conventionally to mathematical comparison. These were 'is shortest', 'is in the middle', 'is highest', 'is the same' (Pasifika boy from low decile school).

Other verbs were non-mathematical, with the largest proportion being static (36%). There was only one example of a mathematical verb. This was 'makes', used to mean equals (Pasifika boy from low decile school).

Comparisons

Despite the limited use of direct verbs of comparison, 21 students did express this concept. This differed by group, with 3 of 12 Pasifika, low decile students expressing comparison, 8 of 12 non-Pacific, low decile students doing so, and 10 of 12 high decile students expressing comparison. There was almost no difference between girls and boys using some form of comparison, so the sexes were combined. The three groups were then compared using a χ^2 test. The probability of this distribution, χ^2= 8.9144, p = 0.0115955. The difference between high decile and both low decile groups was significant at the 0.05 level ($p = 0.034$) but the difference between low decile non-Pacific and low decile Pacific students was not significant, using Fisher's exact test ($p = 0.0559$).

Abstractions versus specific numerical amounts
In these students' answers, there were only a few instances of generalisations like 'some', 'any', or 'much' (8). There were a great many instances of specific numbers (138), which were used as students read the graphs.

Connectives
As stated above, in a story about this graph one would have expected to hear some causal connectives such as 'the next day there was the same amount of money because he didn't add any or take any out'

In fact few Year 4 students used causal connectives. Only 14% of the connectives were causal, primarily either 'because' or 'so'. These were used by nine students, spread across five subgroups. They were used by both boys and girls of all three main subgroups. These students rarely used more than one or two causal connectives.

The largest proportion of connectives (84%) was additive ones, connecting the stories for each column in the graph or part of the story. These connectives were 'and' (54%), 'then' (12%), and 'and then' (20%). Other connectives used, in small numbers, were 'but' and 'when' (1% each). The discrepancy between causal and additive connectives was found in every group of students. There was little correlation between the length of story told and the variety of connectives used. For example, the high decile girls, all told extended stories that had episodes connected by 'and'. Only one of these students used the causal 'so'.

Year 8 students, story type

The explanations told by Year 8 students were more focused, showing a clearer understanding of both the mathematical explanation required and of what happened in bank accounts. Table 6.2 gives the number of students from each group giving various types of answers. As for Table 6.2, only the first column would be correct by the standards set by NEMP, but the second column also were accurate mathematically.

Table 6.2. ***Focus of stories told by Year 8 students for the Bank Account task.***

	Both save & spend or lost	Amount had or description of graph only	Add only	Deduct only	Story without reference to changes on graph	No explanation
Yr 8 Pacific low decile boys	2	2	1	1		
Yr 8 Pacific low decile girls	2	4				
Yr 8 non-Pacific low decile boys	3	1	1			1
Yr 8 non-Pacific low decile girls	1	4	1			
Yr 8 high decile boys	4	2				
Yr 8 high decile girls	3	3				
Totals	15	16	3	1		1

Only five of the 36 students did not describe both the increases and decreases on the graph or give an explanation involving both putting money in and taking it out. As was the case for the Year 4 students, all of these were low decile students. This difference between high and low decile students was significant ($p = 0.01031$).

Year 8 students were significantly better at giving an appropriate answer – either a story closely tied to the graph or a mathematical description of the graph – than were the Year 4 students, at the 0.05 level ($p = 0.01717$).

There were instances in these interviews, as in those for the Year 4 students, where teacher administrators prodded for more of a story when students only gave what happened on the graph.

There was far less difference in the type of stories told by these older students than by Year 4 students, indicating that these students had a clearer idea of what was expected of them, and a better idea of what happened in bank accounts. There was no obvious difference in the type of stories told by students from different ethnic or economic backgrounds.

When the Year 8 students told stories, they were more closely allied to what might have happened in a bank account than were those of the Year 4 students. For example,

> He's ah just got his bank account and he got ten dollars with it, his um mum put in another ten dollars and he's got twenty dollars, but he ah, didn't do all the jobs in his house so um, and he wanted to buy something, he didn't get his pocket money for not doing the jobs in the house so he took some out of his money, and ah he left the his bank account alone for another day, and then he decided he'd that he um, wanted something else which was better than the last one and so he got out some more money.(non-Pasifika boy)

There was one instance of a non-Pasifika girl from a low decile school starting her story with a story-telling convention. This was 'Well, one day Fred…' There was also one instance of a student asking the interviewer, 'Do I have to tell who they are?' (low decile, non-Pasifika girl). This is similar to the Year 4 student who asked 'how do I know if it is a man or a girl?'(girl from high decile school). There were no instances of robbers or of personal stories that reflected the students' families' experience with banks. The instances of stories telling only how much was added or how much was taken out might be accurate, if the graph had been labelled 'Amount deposited in (deducted from) a bank account daily', as would those of the Year 4 students.

Year 8, Linguistic elements

Agent

The agents of students' stories were other people, as they were for Year 4 students, or the columns on the graph. Instances of the use of 'I' were related to personal, conversational statements like 'what should I say?'. There was one instance of 'I' as the agent within the story-telling conventions where a quotation was part of the story.

> he, um, actually um, had ten dollars in the, bank account and, he thought oh, he got, he got his next allowance or pocket money or something, and then like, so, thought well I should deposit ten dollars, (Year 8 boy from high decile school).

All students who gave explanations used a name or pronoun for the person in the story (usually he or they); a pronoun referring to the graph (usually 'it') or an existential agent, as … 'on Monday there was ten dollars…' (Year 8 non-PI boy). When the main agent used by a particular student was analysed, 25 students used 'he', 'she', 'they' or people as agent; 9 used 'it', the day of the week, or an existential 'there' referring to the column height; and 2 did not include an agent in their answer. There was little difference between ethnic or economic groups or genders.

Action

Of all the verbs used, 25% specifically related to money, like 'spent', 15% showed increase or decrease; 18% were non-mathematical action; and 40% were static or sensing verbs. Only two verbs, 'add up' and 'made' for equals, were considered mathematical.

Comparisons

Eleven students made comparisons, using terms such as 'more', 'same' and 'higher'. However, 28 students made comparisons of some sort, using phrases like 'went up', 'put more in', 'it rose' and 'different amounts'. Of the students who did not express comparison directly, some expressed it indirectly through their story, saying what was saved and what was spent. Under both the broad and the more strict definitions of comparison students from high decile schools use this explanation in the majority of cases, but there is little overall difference.

Abstractions and specific numbers

There were 56 instances of abstractions such as 'it', 'any', 'some' and 'much' and 137 instances of specific amounts of money being mentioned. Although the total number of words expressing specific amounts of money is very similar for Year 4 and Year 8 students, the Year 8 students used more indefinite abstractions than did the younger students.

Connectives

In comparison to the Year 4 students, more Year 8 students used causal connectives. In this age group 22% of all connectives were causal. Some students used a variety of causal and additive connectives. The following example includes this variety, and has the connectives in bold.

> ..**So** like for each day or? [Q] Okay, okay...um .. m Monday the amount of money was ten dollars, Tuesday and Wednesday it were quite high, **and** there were twenty dollars, Thursday it wasn't not that great **but** better than Monday, **and** Saturday was the same amount as Monday. [Q]That Tuesday's **and** Wednesday's had the most amount of money in the bank account, and Saturday's **and** Monday's don't have that much money. [Q] Ah, **because** Monday's **like** the start of the week **and** probably from Sunday they probably spent money from their account **and then** spent some more money on Monday **and since** it's Saturday, Saturday's **like** a shopping day and um, they probably went shopping **and** bought quite a lot of stuff **so** they didn't have much money in their bank account, **and** they probably work on Tuesdays and Wednesdays more than they do on Mondays and Fridays. (Year 8 girl from high decile school)

Of the Year 8 students, 5 students from high decile schools, 5 non-Pacific students from low decile schools, and 1 Pacific student from a low decile school used causal connectives in their responses. 7 students from high decile schools used additive connectives only, as did 6 Pacific students from low decile schools. Ten Pacific students used only additive connectives. The significant difference here is between non-Pacific students, both high and low decile, and Pacific students ($p = 0.00536$). Logical connectives are also discussed in relationship to combinations of text elements in the next sections.

Accurate responses and clear language

This task was different to the other tasks in that students were asked to provide a story about a graph. This caused some confusion for many students. It was expected that the students would be able to look at the graph and then describe a logical reason for the change in the amounts held in the bank account. Instead, older students often simply described the amounts of money held in the account on each of the days. Many of the younger students seemed to be completely unaware of what a bank account was but were much more willing to make up a story. This meant that the stories often had little to do with the actual amounts and more to do with robbers. As such, it was not easy to divide students into those with appropriate or inappropriate answers. However, clarity of language could be determined. This can be seen in Tables 6.3 to 6.5.

Table 6.3. *Clarity of language versus type of story/description told*.*

	Clear language		Moderately clear but vague on specific details		Unclear, multiple reruns, vague		Elliptical		total
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	
Save/spend Year 8	1 n-PLD 2 PLD 3 HD	1 HD	1 HD 2 nPLD	1 HG 1 PLD					12
Save/spend Year 4	1 nPLD 2 HD	1 HD		1HD	1HD				6
Graph changes only Year 8	1 HD	4 nPLD 3 PLD 3 HD	2 nPLD 1 PLD	1 HD 1 PLD	-	-	1 nPLD 1HD	1 nPLD	19
Graph changes only Year 4	2 nPLD 2 PLD 3 HB	2 HD		2 HD		2 nPLD 1PLD			14
Only add or only deduct Year 8	1 nPLD 2 PLD	1 nPLD							4
Only add or only deduct Year 4		3 PLD	1PLD 1HD	1 nPLD 1HD					7
Other Year 8	-	-	-	-			1PLD		1
Other Year 4					1 nPLD		3 PLD	3 nPLD 2 PLD	9

*Codes for student identification as in Table 3.1

Table 6.4. ***Summary of accuracy and clarity of language by ethnographic categories.***

	Gender		Year		Decile		Ethnicity	
	Boys	Girls	Year 4	Year 8	High Decile	Low Decile	Low Pacific	Low non-Pacific
Clear language /accurate*	17	14	13	18	16	15	7	8
Clear language / wrong	3	4	4	3	1	7	5	2

*either save/spend or description of graph

Table 6.5. *Summary of unclear or elliptical language by accurate and inaccurate responses by ethnographic categories.*

	Gender		Year		Decile		Ethnicity	
	Boys	Girls	Year 4	Year 8	High Decile	Low Decile	Low Pacific	Low non-Pacific
Unclear or elliptical/ accurate	3	4	4	3	2	5	1	2
Unclear or elliptical/wrong	5	5	9	1	-	10	6	4

These tables suggest that the students most likely to be accurate and use clear language are Year 8 boys from high decile schools, but that the differences between genders and deciles are not strong. Students who used unclear language and were accurate came from all groups, although more came from low decile than from high decile schools. The students most likely to be wrong and have unclear or elliptical language were Year 4 students from low decile schools of either ethnicity.

Among the Year 8 students, girls of all groups were more likely to use clear language when they told a story just based on the graph changes than were boys, but boys were more likely to tell a clear story based on saving and spending than were girls. An example of a girl's story describing the graph alone was the following:

> Umm ... on Monday they have ten dollars, Tuesday and Wednesday they have twenty dollars, Thursday and Friday they have fifteen dollars, and then on Saturday they go back to ten dollars.

An example of a story of save/spend from a boy from a high decile school was:

> On Monday whoever had ten dollars, and then on Tuesday they could have got their pocket money or got paid or something, it went up to twenty dollars they didn't spend any money on Wednesday, then they spent .. oh, is it fif, sixteen .. it looks as if they spent four dollars, so they had sixteen dollars, then they didn't spend any money on Friday, and then they spent some money on Sun, Saturday, which means they had eleven dollars left, or ten, hold on .. yeah ten dollars.

Pacific boys from low decile schools were also likely to use clear language when either giving a save or spend story or when providing an add or deduct story. The following is an example of these types of stories.

> On Monday they only have ten dollars, but on Tuesday it rises, and on Wednesday it still stays the same, then on Thursday, they probably spent a little and it went little, then on Friday stayed the same, and on Saturday it went back to ten dollars.

As can be seen from the last example, the type of story is not always easily categorised, with some containing elements of more than one kind. This last example is considered to be predominantly about the process of getting bigger or smaller but does mention spending as well as making oblique references to the graph, through the use of 'it'.

The types of Premises in the text structures were related strongly to the type of story. The text structures are discussed in more detail in the next sections.

Text Structure

The text elements that the students used in responding to this task were predominantly Premise and Consequence. However, like the Weigh Up task, there were examples of students using other elements such as Conclusions, Elaborators and Suppositions. The different elements are described in the next paragraphs.

Three students used Premises followed by an implicit Conclusion. Two were Pacific girls and the other was a boy from a high decile school. The Conclusions used were implicit: 'that's how much money they got … and that's all'; 'And the large money is twenty dollars .. second one is fif, six, seventeen and a half, that one is ten dollars, it's finished'; and '...Money's going um, it starts at ten and it's going up higher …. Hmm, that's all'. This final clause was added after a comment by the teacher administrator.

Different types of Premises were used in the responses. However, often a range of Premises was used making it difficult to code the responses definitively as was done with the responses to the Motorway task. The following example comes from a Year 4 student.

> Um, there's ten dollars on Monday, twenty dollars on Tuesday and Wednesday, and fifteen dollars on Thursday and Friday .. and, ten dollars on Saturday. Q Um, ah, they might have only got ten dollars and then, then the next day they put another ten dollars onto it .. and then on, then on, then on Thursday, um, someone might have stole some of their money, and on, and .. and on Friday they didn't bring any money in, so then another bank robber came in and stole all their money again.

'There's ten dollars on Monday' was coded as a monetary Premise, as it was based on the money, while 'they might have only got ten dollars' was coded as a personal Premise, as it had a person (although unidentified) as the main actor in the clause. This next example comes from a Year 8 student:

> ...Umm, started off low and then going high and then going down again. Q Um, she's got a raise, and then she got dropped down again.

In the first sentence, all the clauses were coded as 'graphical' Premises, as they are all referring to the graph. However, after the teacher's comment or question, the next sentence contains two personal Premises.

Table 6.6. ***Distribution of students using the majority of one kind of Premise.***

Premises	Gender		Year Level		School Decile and ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
graphical premise	9	10	11	8	7	5	7	19
personal premise	23	22	23	22	14	15	16	45
monetary premise	1	1	1	1	1	1	0	2
personal/ graphical premises	3	0	0	3	1	1	1	3

Table 6.6 shows that even when the texts are categorised according to the type of Premise mostly used, there seems to be no differences between the groups using them according to gender, age, ethnicity or school decile level.

Table 6.7. ***Use of text structures by different groups.***

Text Structures	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
premise	15	17	15	17	11	10	11	32
premise – conclusion	2	1	2	1	2	0	1	3
premise – elaborator – premise	4	1	2	3	3	1	1	5
premise – elaborator + other elements	4	1	3	2	3	1	1	5
premise – consequence	1	4	4	1	1	3	1	5
premise – consequence – elaborator + other elements	3	3	3	3	1	1	4	6
premise – consequence – premise	3	3	4	2	2	3	1	6
premise – consequence – premise – elaborator	3	3	2	4	0	3	3	6
premise – supposition – elaborator + other elements	1	0	0	1	0	0	1	1

Table 6.7 provides information on the distribution by student groups of the different text structures. Almost half of the students gave responses which only included Premises. These students seemed to be evenly distributed across groups. Table 6.8 shows the distribution of students who had a Premise – Consequence combination within their responses. It would seem that, apart from a tendency for students from high decile schools to be more likely to use this combination of elements, there seems to be no clear distinction between the groups.

Table 6.8. ***Text structures with Premises and Consequence.***

Text Structures including:	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
premise – consequence	14	14	16	12	7	9	12	28

In the responses which included a consequence, there were 6 instances of these preceding the Premise, such as ‘and Friday he had a day off because he was sick’. As was the case in responses to the other tasks, these were rare occurrences. The distribution of students using this combination is given in Table 6.9.

Table 6.9. ***Use of Consequences preceding the Premises by different groups.***

Text Structures including:	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
consequence – premise	3	3	3	3	1	4	1	6

Although the number of students who use this combination of elements was small, it was most commonly used by students from low decile schools, and not in Pacific communities. Interestingly, three students joined these clauses with 'because', one student used 'when', while the remaining two students did not use any logical connector.

As with the Weigh Up task, clauses which set up propositions were labelled as Suppositions. In two cases, 'say' was used by students to indicate that the specific example which followed was not necessarily exact. The first example of this came from a Year 8 girl at a low decile school who was not a Pacific Islander. At the end of her response she said, 'and then on Saturday it .. dropped .. to .. say about ..ten dollars.' In the other example 'say' was used to suggest that the series of actions outlined were hypothetical; '....Well, if this is the amount of money, and these are the days, say he came into the bank on Monday, and he like put ten dollars into his bank account then it shows that he's got ten dollars'. Suppositions were more common in the Weigh Up task and are discussed in more detail in that chapter.

The other element in this task was an Elaborator. Most often this was a relative clause which followed a verb to do with a mental action such as 'decide' or 'means', or a relative clause providing extra information about a previous clause or which was embedded within the main clause and provided extra information about an actor or an event. The following is an example from a Year 4 Pacific student attending a low decile school: 'the man that's stealing the money was come every week'. 'That's stealing the money' provides more information about 'the man'. Table 6.10 provides information about the use of Elaborators with Premises and Consequences.

Table 6.10. ***Responses containing combinations with Elaborators.***

Text Structures containing:	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
premise – elaborator	12/36	6/36	8/36	10/36	6/24	5/24	7/24	18/72
consequence – elaborator	6/17	3/17	5/19	4/15	3/8	0/13	6/13	9/34

There were eighteen students who used Elaborators with Premises, either embedded within or following immediately afterwards. Twelve of the eighteen students were girls, but there appeared to be no clear trends between any of the other groups. As well as elaborating Premises, they could also be found less commonly elaborating Consequences. The following is an example from a Year 8 boy from a high decile school: 'he got his next allowance or pocket money or something, and then like, so, thought well I should deposit ten dollars', where 'well, I should deposit ten dollars' is an Elaborator of 'thought'. There were nine students who included an Elaborator with a consequence. Six of these were girls and six were from high decile schools.

The following two tables provide information on the logical connectives which were used in the responses to this task. Given that many of the students doing this task gave a narrative either of someone adding or subtracting money from their bank account or told about the various features of the graph in a logical sequence, it could be predicted that there would be a variety of logical connectives used. This can be seen in the following example from a Year 8 student.

> He's ah just got his bank account **and** he got ten dollars with it, his um mum put in another ten dollars **and** he's got twenty dollars, **but** he ah, didn't do all the jobs in his house **so** um, **and** he wanted to buy something, he didn't get his pocket money for not doing the jobs in the house **so** he took some out of his money, **and** ah he left the his bank account alone for another day, **and then** he decided he'd that he um, wanted something else which was better than the last one **and so** he got out some more money.

However, based on Grice's pragmatic maxim of orderliness, Peterson and McCabe (1991) suggested that 'the act of stating or describing one event before the other is automatically presumed to mean that the event mentioned first actually occurred first' (p. 34). The following tables, therefore, also provide information on the students who do not use a logical connective between a Premise and a Consequence or between two Premises.

Table 6.11. ***Distribution of logical connectives between a Premise and a Consequence.***

Logical Connectives	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
and	5	7	6	6	3	4	5	12
and then	2	1	3	0	1	1	1	3
then	0	1	0	1	0	1	0	1
when	1	1	2	0	1	1	0	2
since	1	0	0	1	0	0	1	1
because	2	2	3	1	1	2	1	4
so	3	5	3	5	0	2	6	8
but	1	1	1	1	0	1	1	2
if	1	0	0	1	0	0	1	1
No LC	7	4	3	8	3	2	6	11

Between a Premise and a Consequence a range of logical connectives were used. The most common one as has been the case in all of the tasks was 'and', which used by a sixth of the students. However, almost a similar number of students used no logical connective at all. Although the numbers are small, the majority were in Year 8 and from high decile schools. This is the group who, because of the closeness of their home language environment to that of the school, were expected to be heavy users of logical connectives, yet about one quarter failed to use any.

Table 6.12. ***Distribution of logical connectives between two Premises.***

Logical Connectives	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
and	26	22	23	25	16	14	18	48
and then	14	14	10	18	5	10	13	28
then	5	9	5	9	4	4	6	14
so	2	2	4	0	0	1	3	4
but	2	3	0	5	1	2	2	5
or	0	1	0	1	0	0	1	1
No LC	10	9	10	9	4	5	10	19

As had occurred in the Motorway task, the use of logical connectives is fairly similar for the groups, with a few exceptions. More students from high decile schools used logical connectives and a larger range of them. They were also the group which chose most often not to use a logical connective between Premises. This suggests that while some students used more than one connective in their response, others chose not to use any at all.

However, there are differences in the use of some logical connectives in a Premise – Consequence combination and a Premise – Premise combination. 'So' as a causal connective is used more often to connect a Premise to a Consequence, but almost always by students from high decile schools and mostly by boys. The four students who used it to connect two Premises together were all in Year 4 and, also, mainly students from high decile schools. Peterson and McCabe (1991, p. 38), in reviewing the literature, suggested that 'narrators use connectives at transition points when they are departing from the timeline of their narratives to insert other relevant information, specifically *so*, *but* and occasionally *because*'. Below are the four extracts in which **so** is used to join two Premises together.

> and on Friday they didn't bring any money in, **so** then another bank robber came in and stole all their money again (Year 4 non-Pacific boy from low decile school)
>
> Q Five dollars, and they leave it there, for five dollars they spend, **so** they don't spend any money on Friday, and they spend five dollars more, so it makes ten dollars (Year 4 boy from a high decile school)
>
> on Monday she had ten dollars, and um **so** on Tuesday and Wednesday she had the same amount, in twenty dollars (Year 4 girl from a high decile school)
>
> on Monday the man, the man went to his bank account and, he had .. ten dollars in his bank account, **so** he, he, he left it to the next day to get some money out (Year 4 girl from a high decile school)

As can be seen from these extracts, none of the 'so's which connected two Premises seemed to fulfil this pragmatic role. The second extract suggests that the boy lost his thought pattern after a teacher utterance and the 'so' marks the point where he picks up the thread of what he is saying again. The other uses suggest that the speakers wanted the following Premises to be Consequences but have not made the connection sufficiently clear. McCabe and Peterson (1985), in research on the use of connectives by different ages of children, also found examples of 'so' where there were no causal links. Some of these were later explained, while others remained unexplained. They found no relationship to the age of speakers. On the other hand, Donaldson's (1986) research into children's use of connectives including 'so', found that logical causality was not mastered until the age of 8 and 9. Given that no similar examples were found amongst the Year 8 responses, it may be that part of the learning process for using 'so' effectively involves a period of time when it is used incorrectly. This would also explain the incorrect use of 'because' in the Better Buy task.

As well as logical connectives, the students used a range of other linguistic devices to suggest cohesion between ideas. One of these devices which was used by many students was the use of an adverb such as 'still' or 'again', or an adjective such as 'another', or 'same', which was used both as an adjective and as a noun. The following table outlines those who used these devices.

Table 6.13. ***Showing the distribution of devices to support the cohesion of the text.***

Cohesion devices	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
still	0	7	4	3	3	1	3	7
again	1	2	2	1	0	2	1	3
even more	1	0	1	0	0	0	1	1
more	7	7	7	7	1	3	10	14
comparatives	5	6	8	3	1	3	7	11
superlatives	6	0	3	3	4	2	0	6
another	1	5	2	4	2	3	1	6
same	11	8	7	12	7	4	8	19

There are certainly significant differences between which groups using these devices. Only boys used 'still' and only girls used superlatives, such as 'shortest' and 'highest'. Students from high decile schools were more likely to use 'more' and comparatives, such as ' bigger' and 'lower', than students from low decile schools. Although the number of examples was small, it would seem that students from low decile schools were more likely to use superlatives than students from high decile schools.

Compared to the Better Buy task and to the Motorway task, where there were differences between the text structures, it is interesting to see that there are no trends for the different groups with this task. This could be partially because the text structures themselves were limited but maybe also because the students were uncertain about what was expected of them in this task. With limited variety in text structures and difficulty in determining what was an appropriate mathematical response, there seemed little point in looking at the text structures in comparison with appropriateness of response. Therefore, this section has not been provided in this chapter.

Hesitant language

The Bank Account task was not readily understood by many students and different groups employed different strategies for expressing uncertainty in responding to it.

Three boys, all from low decile schools, refused to attempt the task at all. Two were from non-Pacific communities whilst the third was a Pacific. Two were in Year 8 whilst the third was in Year 4. Many students also used a range of different linguistic features to mark hesitant language. Many of them would occur in the same utterance, as in the following:

> Umm .. well, here he had like not much money in his bank account then he must of saved a bit more and got another ten dollars in his bank account, and on Wednesday he didn't save any, any money, and he just kept the same in his bank account, then on Thursday he must have spent some, then on Friday he mustn't have spent any, and just left it, and on Saturday he must spent some.

As with the responses to the Motorway task, hesitant language featured hedges, fillers, approximators, questions and disclaimers. The following table provides information on the fillers that students used to gain themselves time.

Table 6.14. ***Showing the distribution of fillers by different groups.***

Fillers	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
umm	21	11	10	22	11	10	11	32
oh/ah	5	7	4	8	3	2	7	12
well	4	3	3	4	0	1	6	7
okay	1	3	0	4	1	1	2	4
hang on, hold on	1	2	0	3	1	1	1	3
yeah	0	2	0	2	1	1	0	2

There was more variety in the fillers that students used in their responses to this task than those used to respond to the Motorway task. However, the use of 'umm' is quite different to how it is used in responses to the Motorway task. In the Motorway task, the numbers were fairly evenly spread between boys and girls, and between Year 4 and Year 8 students. For the Bank Account task, there were twice as many girls and Year 8 students using 'umm' as there were boys or Year 4 students. For the other fillers, the results are similar to those used in the Motorway responses.

Table 6.15. ***Showing the distribution of hedges by different groups.***

Hedges	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
just	2	5	3	4	0	3	4	7
like	5	7	4	8	3	2	7	12
kind of	2	0	2	0	0	1	1	2
probably	2	3	1	4	2	1	2	5
say	2	0	0	2	0	1	1	2
I mean, I think, that meant, which means, lets see, as I say	5	1	2	4	2	1	2	6

The results in Table 6.15 show similarities between the use of fillers and hedges in this task and in responses to the Motorway task, although there are many more students using them in this task. Some of these hedges were used to approximate the amounts that students were discussing. The following table shows the distribution of the approximators used to lessen the exactness of the amounts discussed.

Table 6.16. ***Showing the distribution of approximators by different groups.***

Approximators	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
a little (bit), a wee bit	3	3	2	4	3	2	1	6
like	0	1	1	0	0	1	0	1
some (more) money	3	5	3	5	0	5	3	8
just	2	0	1	1	1	1	0	2
kind of	1	0	1	0	0	1	0	1
a bit more	1	2	2	1	0	0	3	3
lots of money	1	0	1	0	0	0	1	1
quite	2	0	0	2	0	0	2	2
about/almost	4	0	0	4	2	1	1	4

Students used a greater variety of approximators than they did for the Motorway task. This probably reflects the differences in the types of answers expected of them for these two tasks. In this task, students were not given any specific instructions on what details they should include in their graph stories. The requirements for the Motorway task constrained the types of response that students gave thus limiting the types of approximators that they used.

The four Year 8 girls who used 'about'/'almost' did so in regard to the amounts in the bank account on Thursday and Friday which required them to make an estimate against the scale. All the other students who referred to the graph and gave mathematically sensible stories either gave exact amounts or used broad approximators such as 'some'. The following response was by one of these girls who used 'about'.

> Well, if this is the amount of money, and these are the days. Say he came into the bank on Monday, and he like put ten dollars into his bank account then it shows that he's got ten dollars, and he came on the next day and he put um, ten more dollars in, and then he came in the next day he didn't put any money in, because it's the same amount, and then he came in on a Thursday. Lets see, he obviously took out some money ... he took out about .. umm, four dollars, and then he didn't put any more money in on a Friday, and took out some more money...umm .. he took out about six dollars.

As the students were expected to give a story about an imaginary bank account, it was anticipated that many students would use modal verbs such as 'might', 'would'. However, as Table 6.17 shows, this was not the case.

Table 6.17. ***Showing the distribution of modal verbs by different groups.***

Modal verbs	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
might	1	4	3	2	1	2	2	5
could	1	1	1	1	1	0	1	2
must	0	2	0	2	1	0	1	2
would	1	0	0	1	0	1	0	1

The numbers are once again very small. Boys used more of these verb forms than girls, but there seem to be no other major differences between the different groups.

Another way that students showed their uncertainty about what they had to do during the task was through asking questions or stating their uncertainty. Students asked questions, either of themselves or of the teacher administrator. The questions that they asked of the teacher administrator were either about the task itself or about something on the graph. Often these questions could just be a more subtle way of asking for reassurance about the task. This can be seen in the following example:

> Is this how much money he's got in his bank account?

Table 6.18. ***Use of questions by different groups.***

Questions	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low Non-PI	High	
Question of TA about task	5	3	2	6	1	3	4	8
Question of TA about graph	5	2	0	7	5	0	2	7
Question of themselves	0	2	1	1	0	1	1	2

The results for the different groups asking questions are given in Table 6.18. Rowland (2000) reported research which showed that students were reluctant to ask questions because they felt they would lose face in front of their teachers. Similarly, in research looking at Pacific girls in an Auckland high school, Jones (1988) found that these girls were far less likely to question their teachers than their non-Pacific peers. This was not only because of the risk of losing face, but also because they felt that it was disrespectful. It is, therefore, very interesting to find that it is girls rather than boys who are prepared to take such risks, but only once they are in Year 8. There is, however, other research (Langvogt, Leder & Forgasz, 2002) which suggests that girls are less confident with their own ability in doing mathematics and so it could be a lack of confidence which results in them checking what they are doing. Boys, on the other hand, will launch into an explanation even when they are answering a very different question than the one which was asked. This is in contrast to the boys who refused to attempt the task at all. By not attempting it, they cannot be considered to have 'failed' and so resemble the boys in Langvogt et al.'s (2002) research, which suggested that boys thought they failed because they had not studied rather than because of a lack of talent. Girls were more likely to consider that they lacked talent.

As well as asking questions to indicate their uncertainty about what to do, some students expressed their uncertainty about what was expected of them. The following table outlines this.

Table 6.19. ***Use of disclaimers by different groups.***

Disclaimers	Gender		Year Level		School Decile and Ethnicity			Total
	Girls	Boys	Year 4	Year 8	Low PI	Low non-PI	High	
I don't know	3	4	3	4	3	4	0	7
I'm not sure	0	2	1	1	1	0	1	2

These results suggest that boys express their uncertainty in different ways to girls. Women are known to ask more questions as part of their interactional style (Fishman 1978) and, therefore, if boys and girls follow the language patterns of adults, and there is research to suggest that this is true (Deuchar, 1990 and Coates, 1993), it is not surprising to find this also occurs in how boys and girls express uncertainty. However, what is surprising is that this is compounded by socio-economic background. Boys from high decile schools are less likely to express their uncertainty either as a statement or as a question. On the other hand, 4 out of the 6 Year 8 girls from high decile schools asked a question. Boys who did express their uncertainty by either: refusing to do the task; asking a question; or making a statement about it, were all from low decile schools. 3 out of the 6 Year 8, Pacific boys from low decile schools asked a question.

Koehler (1990) reported research which showed that classrooms where girls were encouraged to ask questions resulted in them having poorer results than those in which students were encouraged to be more autonomous in their learning. If, for students in low decile schools, it is acceptable to opt out of participating in tasks, then it may that these students, boys or girls, are not being encouraged to be autonomous learners and this would have an effect on their learning. Alison Jones' (1988) research on Pacific girls found that, compared to their Pakeha peers, they 'avoid[ed] eye contact with the teacher, they spoke up very little, muttered more often and rarely called out an answer as an individual' (p. 148). She felt that these behaviours were reinforced by the demands made on the teachers' time by Pakeha girls thus reducing Pacific girls' opportunities for engaging in ways of behaving which were more likely to produce the learning valued by schools. Although our research suggests that primary students in low decile schools do not express themselves in the ways suggested by Jones, there are differences in how groups of students choose to provide mathematical stories and interact in assessment situations. If these inhibit students' access to the experiences which are most likely to result in them gaining the learning valued in classrooms, these students could be unfairly disadvantaged. More work needs to be carried out to discover how much these different ways of talking affect the learning situations offered to students.

Chapter 7

Stories in mathematics education

Introduction

This section relates to students' responses to two questions from the 2001 NEMP data: Motorway and Bank Account. Both mathematical questions have aspects of stories in them. In the first question, the mathematics appears in a story or in a word problem, and, in the second, students are asked to create a story that relates to the mathematical representation.

The use of word problems in mathematics is well-intentioned, and is an ancient practice. Motivation for using them includes demonstrating that mathematics can arise from real situations, as promoted by Dutch Realistic Mathematics, and is useful for finding solutions to those problems. Experience with genuine problems like planning what food to buy for a class camp while keeping within a budget and meeting nutritional needs, is one way of grounding mathematics in a real problem where the context is never far from students' minds.

There has been a considerable amount of writing on children's suspension of sense making in the face of the conventions of story problems or word problems in mathematics. A common example recently has been the bus problem, in which students are asked how many buses will be needed to carry a group of people from one place to another. This is a problem that students often solve by doing division and leaving a remainder rather than noting that an additional bus would be needed to carry that remainder, or using other real world solutions such as suggesting that only one bus would be needed if it went back and forth often enough.

An army bus holds 36 soldiers. If 1128 soldiers are being bussed to their training site, how many busses are needed?

Figure 7.1. The bus problem.[1]

Research on problems like this has drawn attention to the fact that students are usually asked to suspend their real world logic when attending to mathematics word problems. Nesher (1981, cited in Verchaffel et al.) calls word problems 'a very special type of text'. Verchaffel et al. refer to their 'stereotyped nature' (p. 144). Students learn to respond to them in a manner that looks for the numbers, performs some operation, and may ignore the sense of them. This corpus of literature usually refers to written or textbook word problems. For some of these problems, the student's task is to strip away the unnecessary story aspects, then operate with the numbers in the manner expected, and finally restate the answer in terms of the story

[1] NAEP problem, cited in Silver, Shapiro and Deutsh (1993)

context. Thus 'twenty-seven apples plus thirteen apples' is 27 + 13 = 40, although to give a complete answer students would need to say '40 apples'. For problems such as the bus problem in Figure 2, the task is to operate in a similar manner but the students must consider the context again when giving the answer in terms of buses: not '31 buses remainder 12', or '31.333 buses', but 32 buses.

Operating with mathematics problems in this manner is part of the didactic contract between teacher and student. Both parties have to acknowledge that the words in a mathematics problem are often irrelevant in some manner. Sierpinska defines this didactic contract as,

> The rules and strategies of the game between teacher and the student-milieu, which are specific of the knowledge taught. Sierpinska (1999).

This definition draws attention to the fact that the rules or didactic contract may be different for mathematics than for other subjects, such as creative writing.

Another use of word problems in mathematics is to require students to provide a context in order to make sure that they understand the mathematics given in a numerical problem. One example of this appears in the Chelsea test of numerical operations, in which students are asked to write word problems that fit numerical problems, such as 35 ÷ 5 (Brown, 1981). This is a less common experience in the didactic contract. For this type of problem a fanciful story would be thoroughly acceptable if it fit the mathematical conventions (e.g., if 35 clouds had to be divided among 5 angels, how many clouds would each angel have?).

Difficulties with stories in teaching mathematics may arise because students do not differentiate the expectations for stories in this subject from stories in other school work. In reading, writing, and drama, students are often asked to read, make up or portray a story for the sake of the story itself, not to coat a mathematics problem. They are read stories by their teachers which may be purely fanciful, and may have no underlying meaning other than to entertain. These stories have different conventions to those observed in mathematical problems. They may start with 'Once upon a time', an introduction that immediately cues readers or listeners into the knowledge that a fantasy story is to follow. Characters are to be defined and there is a plot in which these characters interact. A good story builds interest or suspense and has some sort of a climax that precedes the ending, possibly one in which characters 'live happily ever after'. Stories of this type take up more of a young child's day than do the stories invented to coat a mathematical problem. This may set up confusion for students who need to know what type of story is expected in which subject, because the didactic contracts are different.

An additional source of confusion in the didactic contract reflected here was that, although these were school-type problems, they were not presented in a classroom. They were given in an interview with an unfamiliar teacher administrator. Although the interviewer was expected to stick to a script, the nature of the interaction led both parties to interact following rules that were more closely allied to the rules of conversation than to the classroom contract. The conventions of conversation involve a different contract, in which certain types of statement elicit different responses. If the teacher-administrator used a personal pronoun like 'you', the student could legitimately think that she was supposed to respond with 'I'. (see, e.g., Sacks, 1992).

For example, when introducing one of the problems in this study, one about cars going down a motorway, the following conversation took place:

> Interviewer That's a lot more cars than go down the road here.
> Child Yah, it must be in Auckland.

Another child commented,

> If it was Auckland, it would be a typical day

This conversation was appropriate to the adult-child interaction but unnecessary for the mathematics of the problem.

Most Year 8 students appeared to understand the didactic contract for both the Motorway and the Bank Account tasks. They appeared to understand that the didactic contract was different for different types of problems. For the first problem they were expected to adopt the procedure of ignoring the story and dealing with the numbers. For the second problem they were expected to make up a story to explain a graph. However, several Year 4 children displayed confusion about the rules governing the use of stories in these two mathematics problems.

In the first problem, Motorway, they were expected to peel away the story about cars going down a busy motorway in one and nine minutes and attend to the numbers mentally multiplying 9 times 98, and then possibly add the term 'cars' at the end of their answer, although there was no requirement for this. For the second item, Bank Account, they were expected to create a story to match a graph that described the mathematics represented, but despite the fact that the graph was probably fanciful for a student, the story that they made up should not be too fanciful.

One appropriate answer to the Motorway problem was:

> 888 [I: Explain to me how you got your answer] Well, I just went up to 98 plus 2 is 100 and 9 100s are 900 take away… 2 nines… (Low decile non-Pacific boy).

An appropriate answer to the Bank Account problem was one that gave the daily balance and explained deposits and withdrawals.

One appropriate answer to this question was:

> He's ah just got his bank account and he got ten dollars with it, his um mum put in another ten dollars and he's got twenty dollars, but he ah, didn't do all the jobs in his house so um, and he wanted to buy something, he didn't get his pocket money for not doing the jobs in the house so he took some out of his money, and ah he left the his bank account alone for another day, and then he decided he'd that he um, wanted something else which was better than the last one and so he got out some more money. (Year 8 non-Pasifika boy from a low decile school)

Another appropriate answer from a student who was less fluent was:

> And there was ten dollars, then he decided to go back on Tuesday, then he had twenty dollars, then he went back, then he went on Wednesday and he still had twenty dollars...on Thursday, at his work, it went down a little, cos he did a little...what should I say (whispers to himself)..mistake, and Friday he had a day off cos he was sick, and then on Saturday he had ten dollars. (Year 4 Pacific boy from a low decile school)

The conversational conventions of both of these questions require the student to, 'explain' or 'tell me'.

Our interest in relation to the first problem is in students who held to the conventions of story telling rather than stripping these away and responding to the mathematics in these stories. Our interest in the second problem was different, in that students were given a mathematical representation and asked to make up a story about it. It provides another attempt to make mathematics relevant to real life without it actually being relevant, because it is hard to imagine a student with a bank balance that changes daily in this manner. The conventions of making up a story for this situation would be that the story has to relate closely to the graph, explaining each change, and that it should relate events that could make the bar on the graph higher or lower on different days, but, also, that it does not need to contain extraneous information and does not need to start with 'Once upon a time…'

Results

It was largely Year 4 students who failed to understand the didactic contract and story conventions of both problems. Year 8 students seemed to have understood the didactic contract even if they did not answer the problem correctly. Among the Year 4 students, those from high decile schools were more likely to have grasped these unspoken conventions than were students from lower decile schools.
Results for the two problems are presented separately. For the Motorway task, the number of students in each category who gave purely mathematical explanations, stories with an emphasis on the context, or no explanations are presented in Table 7.2.

Motorway

Table 7.2. ***Number of students in different categories giving different types of stories in answer to the Motorway question.***

	Mathematical explanation, with the context added to the answer sometimes	Response with emphasis on cars, not mathematical	No explanation, including guessed, don't know
Yr 8 Pacific low decile boys	4	1	1
Yr 8 Pacific low decile girls	5	1	0
Yr 8 non-Pacific low decile boys	6	0	0
Yr 8 non-Pacific low decile girls	5	1	0
Yr 8 high decile boys	6	0	0
Yr 8 high decile girls	6	0	0
Yr 4 Pacific low decile boys	1	1	5
Yr 4 Pacific low decile girls	1	3	2
Yr 4 non-Pacific low decile boys	3	1	2
Yr 4 non-Pacific low decile girls	4	2	0
Yr 4 high decile boys	6	0	0
Yr 4 high decile girls	4	1	1

Thus 33 of 36 Year 8 students followed the didactic contract, either dealing in mathematics or saying that they did not know. The three who did not give mathematical excuses used proto-quantitative terms, like 'lots of cars' and 'too many cars'. The latter response appeared to come from a student whose first language was not English. Two of these students had an answer that was close to the correct one (810, 950) and the other student was well out (100).

Among the Year 4 students most of the stories were told by low decile students, and more often by girls (6) than by boys (2). All of the students who told stories had incorrect answers. A question arises here: why did some students who were uncertain said that they did not know (10 students) and while some made up stories (8 students)? The students who told stories may have been responding to the conversational conventions covered by Sacks (1992) and trying to please the teacher-interviewer.

Below are the answers that were judged to be stories that did not reflect the mathematical content. They were more elaborate than the protoquantitative explanations given by Year 8 students.

a) I think two hundred and fifty nine.[Q] Because, umm, there's lots of cars going up and down, and um, cars like going visiting and on the bridges. (Yr 4 non-Pacific boy from a low decile school)

b) Hmm ... about … a thousand and fifteen. …[Q] Well, it was by looking at all of those cars, if it take, if, in one minute 94 car, cars go on the road, then there must half, half of it must still be coming, in the nine minutes there would still be some coming. (Yr 4 boy non-Pacific boy from a low decile school)

d) Hmmm [Q] because um...because um, because um people will need to go somewhere to switch cars, but only, um, three eh cars, cars could get there quickly...get to go to get, I mean to go somewhere quickly... [Q]...and...Now, um … I'm riding and some people might get angry, and...sorta people maybe, um, um, some people may can't wait, um, can't wait to the cars um, cars um..go through' (Yr 4 non-Pacific boy from a low decile school)

e) Two hundred. [Q]...Heaps a go, heaps a cars go down the road every day and in nine minutes ...more cars come in (Yr 4 Pacific girl from a low decile school)

f) 460? Could it be? [Q] Because I, if it was nine minutes it could like nine minutes can be like the other cars keeping coming (Yr 4 Pacific girl from a low decile school)

g) Oh, so if 90 would go down in [Q] Like maybe a hundred and something. [Q] Because if there are lots of c(ars), if there are cars go d(own) in 9 minute lots of, because maybe they want to go in a rush, work, for 10 minutes because it might be they have to work at eight o'clock and they only got nine minutes, yah, and because the town is really big and there's a lot of cars that can go (Yr 4 Pacific girl from a low decile school)

h) ....I got no idea, oh because they busy? and they waiting for the stop at traffic light. .Umm...ahumm...[Q] What's that mean? [Q] Eighty nine.[Q] (unintelligible.) because there's many cars there's heaps.[Q] And they're waiting at the traffic not working, won't work and that's all. (Yr 4 Pacific boy from a low decile school)

h) About fifty two.[Q]Yeah [Q] Um...because the motorway's kind of busy and people don't really um hold up on the motorway, because you can kind of, because there basically isn't a speed limit on the motorway, they go quite fast. (Yr 4 boy from a high decile school)

Although four of these answers are given by students of Pacific descent, only (g) gives a story that has evidence of English being an additional language once allowances have been made for the disjointed nature of speech in conversation. This is evident in the lack of a tense marker in 'they busy' and 'they waiting'. It is possible that this boy meant traffic lights when he said 'waiting at the traffic'.

The fact that all of these stories followed either answers or statements indicating that the student didn't know suggests that they did understand the part of the didactic contract that indicated that they were to give a numerical answer. These answers also follow the conversational convention that 'How many…' was to be answered by a number. As they did not know the correct answer or were unable to describe how they achieved it their stories appeared to be an attempt to follow the conventions of conversation. An adult asks you a question after telling a story about cars on a motorway. Therefore it is reasonable to attend the story aspect, especially if you do not understand the mathematical aspect. In a landmark book, John Holt (1964) outlines the strategies that students use when they are not able to complete a problem. When students are aware that they cannot do a problem, they use a variety of strategies to save face. In a direct conversation, this means answering some aspect of the interviewer's question. This can be done by talking about cars, motorways, and people being busy and there being lots of traffic, as is normal in conversation about traffic.

The response of 'don't know' may come from either a confident student who knows the limits of his or her knowledge, or from a student with little confidence. Facial expression and the slope of their shoulders usually distinguish which of these this is. Students' main goal is to get away from the unpleasant situation of being in the spotlight. They use what he calls a 'safe policy' (Holt, 1964, p.5).

Several answers include words that are proto-quantitative, that is, that refer to quantities in an inexact manner. These include: 'lots of'; 'half' (used in an inexact manner to mean some); 'heaps of'; and 'a lot of'. There are several elements in the stories that elaborate on the imaginative context. These children add bridges, visitors, people getting angry and not being able to wait, getting to work by 4 o'clock, traffic lights, and lack of a speed limit. All of these are elements of a story that would do well in creative writing, but are not part of the expectations for a mathematics story.

Bank Account

In contract to the Motorway problem where students needed to strip away the story elements and deal with the numbers, in the Bank Account item they were expected to take an unrealistic mathematical representation and create a story around it. Although this type of request of students is increasingly popular, especially for interpreting graphs, it is still relatively unusual to children who have not learned the conventions of stripping away story elements.

The graph is improbable in children's lives, as they are most unlikely to add small amounts to their bank account or deduct small amounts from it on a daily basis. They are also most unlikely to see a graph of a bank account. The stories of some students indicated that they had seen graphs of the amount of money raised, but that these graphs did not include deductions.

An adequate answer would be:

> Okay, he starts off with ten dollars puts ten dollars in the bank and then he does some work and gets another ten dollars it goes up to twenty, on Wednesday he didn't do anything with it, on Thursday he needs to buy, lunch, or something and it cost him five dollars, and it goes down to .. oh, fifteen, and then Friday nothing, and Saturday he needed to buy something else that cost five dollars and it went down to ten. (Year 8 boy from high decile school)

This would be satisfactory because it mentioned all of the appropriate aspects of the graph, but did not include too much extraneous material.

Table 7.3 gives the types of stories told in response to this question. More categories are used to describe these stories because they differed in more ways. They ranged from stories that only indicated that the height of the bar was different on different days, through to ones that had no more than the necessary story elements, through to ones that still referred to the graph but had many elements. In assigning the stories to types, no attention was paid to accuracy.

Table 7.3. *Number of students in different categories giving different types of stories in answer to the Bank Account question.*

	Stories describe the graph going up and down	Stories with minimal personal elements	Story with superfluous elements	Story without reference to changes on graph	No story
Yr 8 Pacific low decile boys		5			1
Yr 8 Pacific low decile girls		6			
Yr 8 non-Pacific low decile boys	1	5			
Yr 8 non-Pacific low decile girls	2	4			
Yr 8 high decile boys	2	4			
Yr 8 high decile girls		6			
Yr 4 Pacific low decile boys		2	1	3	
Yr 4 Pacific low decile girls	1	4		1	
Yr 4 non-Pacific low decile boys	2	2	1		1
Yr 4 non-Pacific low decile girls	2	2	1	1	
Yr 4 high decile boys	1	5			
Yr 4 high decile girls		4	2		

With this problem it was also the case that the Year 8 students understood the didactic contract better than did the Year 4 students. None of the Year 8 students told stories with superfluous elements or ones that did not refer to the graph. Where they erred, it was on the side of brevity, merely describing the bars of the graph being higher or lower rather than indicating that money had been added or taken out. The brevity of their answers was clear can be seen when they said, for example,

> Well, it's getting um lower um, Monday it's getting higher on Tuesday and Wednesday, lower on Thursday and Friday and Saturday it's lower. (Yr 8 boy from high decile school)

These students reduced their answer to attend only to the essential aspects of the graph, just as students in the Motorway problem had reduced the problem to the essential elements of '98 times 9'.

Among the Year 4 students, six described the changing length of the bars and 19 students told minimal stories. The 19 students who told minimal stories adhered to the didactic contract for this type of problem. Many of these stories indicated that they did not understand that it was the balance in the account that was changing, but that is not the focus of this chapter. Of interest here are the 10 students who told stories that went well beyond the demands of the question and sometimes disregarded the graph altogether. These students all came from Year 4 groups except the boys from high decile schools, whose answers were more like those of Year 8 students. These six boys understood that the question required describing the mathematical representation only. The other groups of Year 4 students were less certain about these rules. Twelve Year 4 students, spread across all groups except high decile boys, relied more on story-telling conventions, as would be expected in school in their writing and reading, and possibly out-of-school settings.

The stories with no reference to the graph were:

a) Hmm....the people come in with all their money. [Q] Hmm...the whole city gets to every single banks.[Q] Banks.[Q] They got too much money inside them. (Yr 4 Pacific boy from low decile school)

b) Oh, um, when my dad gets paid, yeah, he'll put money in my bank for me. (Yr 4 Pacific boy from low decile school)

c) The man, the man that's stealing the money was come every week.[Q] Cos, m….[Q] Oh, the man comes and takes the money every week, and, and, then the bank account doesn't have any more money.[Q] Because the man took the rest of the money. (Yr 4 Pacific boy from low decile school)

d) Um, some of them...um...someone's been borrowing it.[Q] Um....um...they used it because they had no money.[Q] Hmm...they borrowed it because they needed to buy food with it.[Q] Hmm (shakes her head), no...they needed to buy um...lunch for um, their daughter's um, oh .. lunch, for school. (Yr 4 Pacific girl from low decile school)

e) Well that person might take the money.[Q] Um...I don't know.[Q] Why? [Q] Yeah, mum hates going to the bank, to get some power. (Yr 4 non-Pacific girl from low decile school)

All of these stories indicate that the students knew that banks were related to money and that money goes into banks and is taken out of them, usually by other people. The numbers of questions asked by the teacher administrators indicate the difficulty that they had in getting students to tell these stories. However, they do not indicate an understanding of a graph or even that it is the graph rather than the bank that is expected to be the subject of the story. Several of the answers do have the elements that suggest personal involvement in the story that they make up.

Three of the stories that included unnecessary elements were those below. For the sake of brevity, only the extra elements are included.

a) …, and Friday he had a day off cos he was sick, and then on Saturday he had ten dollars..[Q] Cos he spent five dollars, then on Sunday there was the weekend off. (Yr 4 non-Pacific boy from low decile school)

b) … then on Thursday, um, someone might have stole some of their money, and on, and .. and on Friday they didn't bring any money in, so then another bank robber came in and stole all their money again. (Yr 4 non-Pacific boy from low decile school).

c) Um .. the robber is .. tooken .. fifteen dollars.[Q] They took twenty five dollars.[Q] .The same, on Tuesday.[Q] They took .. twenty dollars. …. (Yr 4 Pacific boy from low decile school)

These stories, which all referred to the graph, had some of the same story elements of the stories above that did not refer to the graph at all. They included robbers and a man who was not paid because he did not work.

They differ from two other such stories told by upper decile girls that are given below. These two stories indicate that these girls were well versed in the rules of story telling. These rules or conventions appeared to overrule the need to describe what had happened as displayed in a graph. The videotape of one of these transcripts below shows the girl looking into space or at the interviewer as she tells her story, rather than referring to the graph. She does start by referring to the graph, but after this initial reference to money being put in a bank, she is not concerned with the details of the graph. The teacher-administrator goes beyond the intended script in attempting to draw her attention back to the graph. The comments of both the girl and the teacher-interviewer are, therefore, included.

> Girl: Uhmm…Once there was a little boy called James and he had lots a money and didn't know what to do with all of it, so he decided on Wednesday afternoon he'd go to the bank and put it in his bank account.
> I: Okay, How much money did he have in the bank account on Wednesday afternoon?
> Girl: Twenty five dollars?
> I: What happened to the money for the rest of the time?
> Girl: Just stayed there in the bank.
> I: Does the graph tell you that it stays there in the bank?
> Girl: No.
> I: Or does the graph tell you …?
> Girl: It tells you...how much...no...it tells me how much money down there, not just (unintelligible) and, yeah.
> I: Okay, Can you tell me a little bit about the money that's in this bank account?
> Girl: Here's only twenty five dollars out of the whole lot of that.
> I: Okay
> Girl: And...it's kinda, it's kind of a little bit like a pattern, that's small and .. um, yeah.
> I: Anything more you would like to add?
> Girl: (shakes her head). (Year 4 girl from a high decile school)

The other girl from a decile high school attended to the graph much of the time but also looks into space and smiles while adding details like buying sugar and cherries. She smiles at the teacher-administrator when adding some details, as though she is pleased with her creativity.

> Oh, like do you...like do you, um, make a story up? [Q] ....But how do you know if it's a man or a girl? [Q] On day on Monday the man, the man went to his bank account and, he had...ten dollars in his bank account, so he, he, he left it to the next day to get some money out, and on Tuesday he had twenty dollars, so he um, twenty, thirty dollars altogether, so he, he got his ten dollars out and he went to the store and bought some sugar and some cherries, and then he, on Wednesday had twenty dollars as well, um…um...so he had forty dollars left...um....he left it in his bank account to save up, so he, he …said 'oh I'll just look through all my money, so he's and see how much I've got, so on Thursday, um, Thursday, um, he looked how much money he had, he had um...fifteen dollars, um, and that mean, meant he had...fifty-five dollars, and then he, he went to the store, the bank account the next day, and, and then he looked, um, how much money he had, and he had fifteen dollars again, and so he had seventy dollars, and then on Saturday he went there and he saw, he saw that the bank got robbed and his money wasn't there. (Year 4 girl from a high decile school)

These last two stories include several story-telling conventions that are not usually included in mathematics stories. The first student shows that she knows how to start a story, with ‘Once there was’, an equivalent of ‘once upon a time’. She establishes her character and begins the plot with some background. If she had not been interrupted by the interviewer, it is possible that she would have told a complete story without reference to the specific nature of the graph. These extra questions break the train of story telling, but do not result in an accurate story for the graph.

The second girl needs first to establish who the characters in her story are. Having satisfied herself on this, she then starts her story. Her story is basically one long sequence with separate episodes joined with ‘and’ or ‘so’, but these lead to a surprise climax on Saturday when she discovered that the bank had been robbed. It even includes direct speech by its character. It includes mathematical misconceptions, such as an apparent belief that the money on each day is not the total but the amount that mysteriously arrives in the account. The story would be likely to earn praise and receive full marks in creative writing or story telling, but fails as a mathematical description.

Stories of the students from lower deciles included suggestions of personal poverty such as ‘My mum hates to go to the bank, to get some power’. Others reflected communal ownership of money in money owned by ‘the people’ (P044-C2, P023-C3). This could reflect church or other money raising activities in which communities worked together. Several students associated the graph with only deposits or only withdrawals. In one case, this was a story of money being raised, a situation that they would have been more familiar with than with a graph of deposits and withdrawals.

These students responded primarily to the aspect of the question that required them to tell a story for the graph. Few appeared to have experience with a bank balance that changed daily. The agents in their stories were other people – the man, the people. One student asked, ‘how do I know if it is a man or a girl?’ Only one student used ‘she’ as person responsible for the changes.

Summary

Why did the students quoted above tell stories that were not closely related to the mathematical issue to hand? For both questions, the younger students told these stories while few of the older students did. Thus it is possible that they did not know the mathematics required and possible that they did not understand what was required of them under the didactic contract. In the absence of this knowledge, they responded to the rules of conversation and answered the interviewer literally, giving a story that would be appropriate in conversation.

In the case of the Motorway question, all students who told stories had incorrect answers. This was also true for the Bank Account answers that did not refer to the graph and for some of the stories with extraneous material in them, but this was not always the case.

Seen through the lens of the didactic contract, it appears that older students clearly understand the contract for both types of mathematical stories. They were aware that the stories are unnecessary shells that can be discarded in the case of the Motorway problem and kept to a bare minimum in the case of the Bank Account story. Among younger, Year 4 children, the high decile boys were most aware of the requirements of the didactic contract. They may have had an advantage in that they were less interested in verbal fluency which could have interfered with succinctness. In both the Motorway problem and the Bank Account problem, the groups that told the most irrelevant stories were low decile, Pacific students, followed closely by non-Pacific low decile students. For both problems, they told stories in cases for which they did not know the answer but did know the conventions of story telling or at least had some personal experience to share. This sharing often came after repeated prompts or probes. Stories with extra elements, like robbers or what the money was needed for, occurred in answers that were correct, although these extra elements were not needed for the didactic contract in a mathematical context. Their creativity was a pleasure to see. It is rather a pity that, from the evidence of Year 8 students, this creative flair is likely to disappear in a few years time.

Chapter 8

Text structures in mathematical justifications and explanations

Mathematical explanations and justifications have an important role in mathematics assessment (Bicknell, 1999). Yet, as was discussed in Chapter 1, it would seem that there is limited knowledge about what constitutes good examples of these. Part of the issue is knowing what the constituent parts of explanations and justifications are and how the differences between them are expressed. In this research, explanations were considered to be the description of *what* was done in solving a problem; and it had been expected that students would provide explanations when responding to the Weigh Up and Motorway task. Justifications were defined as the descriptions of *why* a certain strategy was adopted. The Better Buy and Bank Account tasks were expected to illicit justifications. Such a clear cut distinction was not found, especially when students struggled with the purpose of the task, as was the case with Bank Account.

There was also a need to research the stages that students went through as they learnt how to produce a mathematical explanation or justification. Primary school children are not expected to produce mathematical proofs, which have been described as 'perhaps the ultimate in justifications' (Sowder & Harel, 1998, p. 670). Therefore, it was postulated that there might be series of linguistic features that students would need to learn as they progressed through school, so that, by the time they reached university they had mastered all of the features needed to construct a mathematical proof. It was anticipated that research on the responses from these tasks might contribute to understanding what these intermediate features might be. The differences found between groups in how they used these features would suggest that there is no clear pattern of mathematical language acquisition. Instead, what language is acquired and valued within certain situations is affected by a large number of issues. This chapter discusses in detail differences between text structures and the issues which seem to affect them, including the demographic characteristics of the students and the task requirements.

First, it is worth reviewing what was already known about explanations and justifications. This research can be divided into what has been done at the macro or micro level of analysis. At the macro level, Krummheuer's (1995) work, based on the ideas of Toulmin (1969), on the components on mathematical arguments has been used by several other researchers, such as Yackel (2001) and Forman et al. (1998). These components were: claims, which were the suggested solutions; grounds, which were the unchallengeable facts from which the claims were developed; warrants, which were the information joining the grounds to the claims; and backings, which provided the constraints under which the warrants were valid. These researchers used these ideas to discover how students working together and with the teacher developed 'collective argumentation' (Forman et al., 1998). They were not looking so much at how individual students would produce explanations. Other macro level analysis was undertaken by Sowder and Harel (1998), which looked at high school and college students' responses and classified them according to the evidence used in them. These were: externally based proof schemes; empirical proof schemes; analytic proof schemes. However, this macro level research did not provide information on the linguistic structures which could be related to these components of arguments or types of justifications.

The micro level research on students' explanations was that done by Bills (2002 and Bills & Grey, 2001). This research is considered micro level as it documented the linguistic features which seemed to be related to the accuracy of students' explanations as to 'what was in their head' (Bills, 2002, p. 100). Students who used 'I' or 'you' in a general sense, logical connectives such as 'because' and 'if' and present tense were more likely to give an accurate response. Typical classroom talk, such as using 'you', Bills (2002) hypothesised, was more likely to be used when students were successful. Certainly some of these linguistic features were evident in the responses to the four tasks, particularly the use of logical connectives and the use of personal pronouns. However, Bills did not relate these linguistic features to the different components of the explanations.

By identifying the different text elements and the typical ways in which they were combined, we have been able to analyse the responses at both the macro and micro level. This builds on the work of Hasan (Halliday and Hasan,1985) who described how contextual configuration could be used to predict the text structure. As well, the reverse is also true, in that the text structure could be used to illuminate the contextual configuration. Hasan's (Halliday and Hasan, 1985) ideas about text structure require not only an identification of the different text elements, but also the recognition of what elements were compulsory, whether the positioning of elements were set and whether elements could be reiterated. This then allows for an investigation of linguistic features within the elements in order to attain a better understanding of these elements and how they are placed together to form the text structure.

Contextual configuration and text elements

For all of the tasks, the contextual configuration was that students were in an interview situation at school with a teacher administrator who was asking questions to ascertain their mathematical knowledge. The particular knowledge being assessed was slightly different in all cases but all required the students to provide insights on how they perceived the mathematical relationship between objects, mathematical or physical (numerical and quantitative comparisons). It was therefore possible to see how the similarities in the text elements used across tasks was related to the general situation in which the student was operating. By looking at the differences between the responses for the tasks, it was possible to see how the task specifications as part of the contextual configuration influenced the text structures.

It was quite clear that no elements were used by every student in every task. However, there were several elements which were used by many students in more than one task. Their use across tasks suggested that they were likely to be the text elements used consistently by students in responding to any mathematical assessment task orally. These elements were Premise, Consequence, Conclusion, Elaborator and Supposition. Table 8.1 shows the number out of a total of 431 students who used each of these text elements. In this table, the three sections of Weigh Up are considered different tasks, but there were only 71 samples for the Explanation section of the Weigh Up task. As the samples for the two 1997 tasks were of students from low, middle and high decile schools and the two 2001 tasks had students from only low and high decile schools there are problems in combining these results. In order to give an overview of which elements were used most often, the two groups of students, Pacific and non-Pacific, were totalled together and this amount then halved. No amounts were recorded for students from middle decile schools, and so amounts in the low and high decile columns do not add up to the total amounts.

Table 8.1. ***Use of text elements by different groups.***

Text Elements	Gender		Year Level		School Decile			Total
	Girls	Boys	Year 4	Year 8	Low		High	
introduction	9	9	10	8	8		6	18
premise	206	197	197	206	132		140	403
consequence	127	130	117	140	74		99	257
physical consequence	48	43	45	46	32		30	91
conclusion	46	49	38	57	22		41	95
supposition	26	16	13	29	9		13	42
elaborator	92	79	75	96	50		67	171

This table clearly shows that Premises were the most common text element used, with Introductions being the least common. There were few differences between groups using these elements. However, it would appear that girls use Suppositions and Elaborators more often than boys. It would also seem that Year 8 students tended to use more elements than Year 4 students, although the amount of difference varies for the different elements. Year 8 students used more Consequences, Conclusions, Suppositions and Elaborators. Students from high decile schools also used more Consequences, Conclusions and Elaborators than their peers at low decile schools.

Within many of these elements, there were variations. The next sections look at each of these elements and their variations, before looking at how they were combined. The following sections show that very rarely did differences between groups persist across tasks, indicating that the tasks as part of the contextual configuration had a significant influence on the text structures.

Introduction

This element was used by only a few students in responding to four tasks. These were Motorway and the Weigh Up Plan, Description and Explanation. Introductions set the scene for a personalised or generalised response to the task. Examples of these Introductions were 'I went' or 'you could'. Given that most of the scripts for the tasks have the teacher administrators saying 'Tell me how' or 'Explain to me,' it is, perhaps, more surprising to find that so few students began their responses by acknowledging such a request with an Introduction. It would seem that in some tasks a Premise - Elaborator combination was used by many students to fulfil the same purpose. An example of this combination would be 'You'd tell them to use one of these' which was said by a Year 4 student in responding to the Explanation section of the Weigh Up task. Table 8.2 sets out who used Introductions.

Table 8.2. ***Use of Introductions in different tasks.***

Introductions	Gender		Year Level		School Decile			Total
	Girls	Boys	Year 4	Year 8	Low		High	
WU Plan	2	1	2	1	1		1	3
WU Description	2	1	3	0	3		0	3
WU Explanation	0	1	1	0	0		1	1
Motorway	5	6	4	7	4		4	11

Even with such small numbers of students, there does not appear to be any distinction between the groups who used Introductions. Motorway was the only task where more than 10% of students used an Introduction.

Premise

In explanations and justifications, there is a need to provide evidence which would convince others (Sowder & Harel, 1998). The Premise is the linguistic embodiment of the basis for this evidence and, therefore, can be considered equivalent to Krummheuer's (1995) warrant. Certainly, as can be seen in Table 8.1, Premises were used by most students in most tasks, with little difference between groups. However, as can be seen by the reduced number of low decile students who used Premises in their responses to the Description part of the Weigh Up task, the task requirements have an effect even on this fundamental component. In these responses, Physical Consequences were used more often than Premises, as students had to describe their physical actions. These physical actions may have become the unspoken Premises.

Table 8.3. ***Use of Premises in different tasks.***

Premises	Gender		Year Level		School Decile			Total
	Girls	Boys	Year 4	Year 8	Low		High	
Better Buy	36	36	36	36	24		24	72
WU Plan	35	31	31	35	22		23	66
WU Description	27	27	25	29	16		21	54
WU Explanation	36	34	34	36	23		24	70
Motorway	36	36	36	36	24		24	72
Bank Account	36	33	35	34	23		24	69

Although Krummheuer (1995) suggested that warrants were unchallengeable facts, this was not the case with all Premises. Different types of Premises reflected the type of evidence which was given and can, therefore, be related to the proof schemes classified by Sowder and Harel (1998). In the responses to the Better Buy task, there were hypothetical or factual Premises. In the Motorway task responses, Premises were divided into factual, mathematical and personal. For the Bank Account task, there were personal, graphical or monetary Premises in the responses. In the Weigh Up task, there were no clear distinctions in the Premises which were used, as most students started their responses by relating an action that they would do or had done to one or more of the boxes.

Factual Premises were in the responses to both the Better Buy and Motorway tasks and used information given in the question or in a resource. In the Better Buy task, factual Premises were more likely to be used in inaccurate responses. Students who gave them were less likely to combine them with Consequences and Conclusions to produce a train of reasoning. On the whole, it was more likely that students who gave factual Premises were girls, in Year 4 and attending low decile schools.

Responses to the Motorway task showed a similar relationship as only four students followed a factual Premise with a Consequence whereas 13 students followed a mathematical Premise with a Consequence. This would suggest that students saw the most convincing evidence as deriving from an external source, as was also the case with the externally based proof schemes described by Sowder and Harel (1998). Only 4 out of the 20 students, who used a factual Premise, gave an accurate answer for this task. It seems that if students used this type of Premise, they were unlikely to develop them into a logical reason. In this task, the students who were most likely to give a factual Premise were Year 4 girls from low decile schools. There did not seem to be any differences in the ethnicity of the students who gave factual responses.

As factual Premises used the unchallengeable facts, from those given in the question or in a resource used with the question, they could be considered grounds for a mathematical argument. However, as they were less likely to be used in an extended response, the links from them to their corresponding claims were too tenuous for clear mathematical explanations or justifications to be considered to have been given. This was the case for the Year 4 Pacific girl who gave factual Premises in the following response:

> ...Heaps a go, heaps a cars go down the road every day and in nine minutes .. more cars come in.

When the distribution of students in Better Buy and Motorway who used factual Premises is considered together, it would seem that Year 4 girls from low decile schools were most likely to use these Premises.

Hypothetical Premises were the other type of Premise used in responses to the Better Buy task, but were more likely to be found in accurate responses. These Premises proposed an example as the basis for their justification and so could be classified as providing an empirical proof scheme (Sowder & Harel, 1998). They began with the logical connective 'if' and were most often followed by Consequences and then Conclusions, so that logical trains of thought were developed. Of the 25 students who used these Premises, 18 were in Year 8 but only 5 were from low decile schools. There did not appear to be any difference due to gender. In the Explanation section of the Weigh Up task, 25 students also began their Premise with 'if', suggesting a similarity with hypothetical Premises. Certainly, it would seem that in using 'if' students marked that an example was being proposed as an aid towards a general solution. It was interesting to note, however, that the differences between the groups found in the responses to the Better Buy task were not evident in the Weigh Up task. It is difficult to know whether this was due to only having a sample size of 72 for each task, or whether the context drawn upon for each task affected different groups' choice of linguistic features.

Mathematical Premises in responses to the Motorway task were the only Premises to be given in accurate responses. However, unlike the hypothetical Premises, they did not have to be combined with a Consequence to be considered an accurate and clear response. For example, six Year 8 students, four from Pacific communities, were able to provide just a mathematical Premise and be considered correct. Given that students are asked to 'Explain to me how you got your answer', providing a mathematical Premise such as 'Nine minutes times ninety-eight cars' is acceptable. It gives all the necessary information to answer the request appropriately. As mathematical Premises use mathematical calculations as their evidence for their solutions, they have similarities with factual Premises, as they draw on externally based proof schemes. Out of the 42 students who gave a mathematical Premise, 29 were Year 8 students. Although there would seem to be slightly fewer Pacific students who gave mathematical Premises (9 out of 42 students), there do not appear to be any gender or decile level of school attended differences for non-Pacific students.

Mathematical and hypothetical Premises were mostly used in accurate responses and were usually combined with Consequences and Conclusions. In considering them together, they appear to have been used mainly by Year 8 students with no distinction according to gender. Distinctions according to decile level of school attended occurred with hypothetical Premises, but, in response to the Motorway task, the distinction was between Pacific and non-Pacific students. However, there were differences in the type of proof schemes produced. Hypothetical Premises were used with Consequences and Conclusions to produce empirical proof schemes, whilst mathematical Premises were developed into externally-based proof schemes. It would seem that the task requirements influenced the type of Premise necessary to achieve an appropriate response. It would also seem that not all students were able to recognise what was the most appropriate Premise to use in responding to the task.

Personal Premises were used in responses to both the Bank Account and Motorway tasks. These Premises did not occur in accurate responses and did not link to any kind of proof scheme. In the Motorway task, personal Premises were most often mental processes such as 'guessed', 'think' and 'said'. In the Weigh Up task, 'think' was classified as a Supposition and was often used to lessen the certainty of what was being proposed. In responses to the Motorway task, personal Premises were more likely to be used to express uncertainty about what they had to do. Only one student combined a personal Premise with a Consequence. Students who gave these Premises were most likely to be boys in Year 4, from Pacific Island communities and attending low decile schools.

If these results are considered with those for factual Premises, which also tended to be used in inaccurate responses, it would seem that these types of Premises were mostly used by Year 4 students. This probably reflects the fact that these students did not have sufficient mathematics to respond successfully to the tasks. Whereas girls tended to use factual Premises which repeated information from the question, boys, in responding to the Motorway task, tended instead to use a personal Premise. It would be interesting to know whether teachers felt that factual Premises showed that students knew more mathematics, as they often included mathematical information from the question. When examining responses for a similar NEMP mathematics task, Anthony and Walshaw (2002) found that, when students repeated parts of the question, these were accepted as explanations by the teacher administrators as no further probing was done. This may mean that boys, in being blunter about their lack of knowledge, may be perceived as knowing less and thus being given more support to improve their mathematical knowledge.

In responses to the Bank Account task, personal Premises were clauses which had a person, often unspecified, as the main actor. These personal Premises were, therefore, of a different kind from those used in the Motorway task. In responses to the Bank Account task, more students used these Premises than any other kind, but more often than not they were combined with other kinds of Premises within the one response. There were no differences in the groups of students who gave this Premise. Graphical and monetary Premises were also specific to responses for the Bank Account task, as they related to the graph provided as a resource or to money respectively. The responses to the Bank Account task were the least complex and often were a series of personal, graphical and monetary Premises. Sometimes, they were combined with Consequences. As with personal Premises, there were no differences in who gave these types of Premises.

Premises can be considered the grounds for an argument, but were most likely to be used in appropriate responses if they were hypothetical or mathematical. It was less likely that, if a student used other types of Premises, they would be combined with Consequences and Conclusions into complex explanations and justifications. This was because the evidence which they provided needed to be a strong foundation from which a logical train of thought could be developed. Although it would have seemed that factual Premises based on information supplied in the question or in a resource would also have provided this basis, these Premises were rarely combined with Consequences and Conclusions and used in appropriate responses.

Consequences

If Premises of a particular type can be considered grounds, then Consequences have the potential to match Krummheuer's (1995) warrants and backings, as they could contain the information that relates the grounds to the claims. In order to produce an acceptable mathematical explanations and justifications, it is necessary for the grounds to be manipulated in some way, so that the logical relationship to the claims is clarified. This includes identifying the conditions under which the manipulation is valid. Esty (1992) discussed the use of logical connectives in identifying the conditions for which certain mathematical statements were true. In a general sense, Consequences can be considered warrants and the way they are connected to their corresponding Premises can be considered the backings. The conditions for when a manipulation is valid is often contained within the logical connectives joining the Premises to the Consequences, as well as within Elaborators connected to Consequences. In this section, the logical connectives used in a Premise – Consequence and Consequence – Consequence are also discussed.

Consequences were of two kinds; logical and Physical. Logical Consequences always described the result of a manipulation of Premises and were never used by themselves. Table 8.4 outlines who used logical Consequences in their responses to the various tasks.

Table 8.4. ***Use of Consequences in different tasks.***

Consequences	Gender		Year Level		School Decile			Total
	Girls	Boys	Year 4	Year 8	Low		High	
Better Buy	18	20	10	28	9		13	38
WU Plan	20	26	24	22	13		17	46
WU Description	25	25	22	28	15		19	50
WU Explanation	35	31	32	34	21		23	66
Motorway	15	14	13	16	7		16	29
Bank Account	14	14	16	12	9		11	28

If Consequences are considered to be warrants, then it is quite clear that students felt that the explanations for some tasks required them more than others. As students moved through the Weigh Up task, more began to use Consequences so that when they gave their final explanation only five students did not use a Consequence. In regard to the different groups, there appears to be no difference in use according to gender. On the whole, Year 8 students used more Consequences than their Year 4 peers, but most of this difference is accounted for in the responses to the Better Buy task. More students attending high decile schools also used Consequences, but there were no very large differences for any particular task.

In almost all tasks, Consequences combined with certain Premises were linked to responses which were considered accurate and clear. For example, in the Better Buy task, responses which combined hypothetical Premises with Consequences and Conclusions were often accurate. It was more than likely that, in responding to this task, it was a Year 8 student not attending a low decile school who used Consequences. As well, in responses to the Motorway task, there was no difference according to gender in who used Consequences, but there was also little difference between Year 4 and Year 8 students. As was seen in Table 5.6, although students attending high decile schools were slightly more likely to use Consequences than non-Pacific students, both groups were far more likely to use them than Pacific students attending low decile schools. In the Weigh Up task, there were slightly more boys who used a Consequence in their response to the Plan section. However, there were no other differences between groups using Consequences in this section. In the Description section, slightly more Year 8 students used Consequences than Year 4 students. In the responses to this section of the task, there were no differences in gender, ethnicity or school decile level. In the responses to the Explanation section of the task, there were no differences between groups who used Consequences in their responses.

The distribution of logical connectives between Premises and Consequences and between Consequences suggested that the task requirements influenced their use. When a Premise began with an 'if', as was the case for hypothetical Premises, it was unlikely that another connective would be provided in front of the following Consequence. However, in long responses, such as those given for the different sections of the Weigh Up task where Premise – Consequence combinations were iterated, some students would often use several different logical connectives to join Premises and Consequences and groups of Consequences together. On the whole, it was more common for students to use narrative rather than causal connectives. However, some tasks, such as Motorway, seemed to encourage students to use more causal connectives than others. This may be because the questions in this task asked more explicitly for an explanation than the Bank Account task did. Those who were more likely to use logical connectives also seemed to change depending upon the task. When all the connectives are considered together, it was more likely that the students who used them would be in Year 8 and more probably not be attending a low decile school or not be a Pacific Islander. However, these distinctions were not always clear cut across tasks, with more Year 4 students using more logical connectives when responding to the Plan section of the Weigh Up task.

Physical Consequences were the other type of Consequences. They could be found without Premises and were used mainly in responses to the Plan and the Description sections of the Weigh Up task. They described the result of a physical manipulation of the boxes and so the action itself can be considered as a silent Premise.

Table 8.5. ***Use of Physical Consequences in different tasks.***

Physical Consequences	Gender		Year Level		School Decile			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
WU Plan	11	10	11	10	10	4	7	21
WU Description	31	32	31	32	20	21	22	63
WU Explanation	6	1	3	4	2	4	1	7

As Table 8.5 shows, there was little difference in those who used Physical Consequences in the three sections of the task. As it was the same students who were giving explanations in all three sections, it is quite clear that the task requirements had a strong influence on the inclusion of these text elements. The Description section of this task required students to talk about what they were doing as they were doing it. In a shared situation where the students were physically manipulating the boxes, it is not surprising to find that Physical Consequences were the most common elements in responses.

Sometimes Physical Consequences were followed by logical Consequences. In responses to the Plan section of the task, 21 students gave a Physical Consequence, but only three followed it with a logical Consequence. However, the situation changed in the Description part of the task. Of the 63 students who used Physical Consequences, 36 followed them with logical Consequences. 94% of accurate and clear responses to the Description part of the Weigh Up task contained Physical Consequences combined with logical Consequences. A considerable number used causal connectives between them, with 25 students using ‘so’ and a further three students using ‘so then’. More causal connectives were used in this position than in any other connections between text elements. These causal connectives were most likely to be used by girls, Year 8 students and least likely by students attending low decile schools. Although fewer students attending low decile schools had used logical Consequences, the difference in causal connectives is larger.

Consequences are important features in mathematical explanations and justifications, as they fill in a step from relating the information contained in the Premise to the actual claim being made. As such, they can be seen as fulling the role of warrants. However, in joining logical Consequences to Premises, it would seem that if a causal connective was used, it would be ‘if’ and would occur before the Premise. On the whole, narrative connectives such as ‘and’ were most often used between Premises and logical Consequences. When Physical Consequences were used in responses to the Description part of the task, a large number of students used the causal connective ‘so’ to join them to logical Consequences. Research with responses to other tasks which require physical manipulation of materials needs to be done to see whether the use of causal connectives is more likely in this situation. Although Consequences could be considered equivalent to warrants, when considering how backings are linguistically represented the use of narrative rather than causal connectives limits the information about when the warrants were valid. Backings are also linguistically represented through Elaborators, as they also provided extra information about Consequences.

Conclusions

Conclusions were text elements which were not found by themselves. On the whole they rounded off an explanation, often coming after a Premise and a Consequence. They are, in many ways the linguistic equivalent of Krummheuer’s (1995) claims, as they are the outcome of the explanation or justification. As can be seen in Table 8.6, they were used in responses to all tasks, although it was only in the responses to Better Buy that more than half the students used Conclusions. This is not as surprising as it first appears. The scripts for Better Buy and Motorway both initially ask students for a result of a calculation before going on to ask for a justification and an explanation, respectively. Therefore, a repetition of this result is not necessary for the explanation to make sense. Yet, in explaining why one box is better value for money than the other, it is not unreasonable to include a second mention of the actual result. As well, some students pre-empted the second question in this task and so the Conclusion came at the beginning of the explanation as well as at the end.

Table 8.6. ***Use of Conclusions in different tasks.***

Conclusions	Gender		Year Level		School Decile			Total
	Girls	Boys	Year 4	Year 8	Low		High	
Better Buy	19	25	15	29	8		18	44
WU Plan	0	2	1	1	0		2	2
WU Description	2	1	1	2	1		1	3
WU Explanation	16	10	11	15	8		9	26
Motorway	7	10	8	9	4		10	17
Bank Account	2	1	2	1	1		1	3

In the Bank Account task, there was no common way for completing the story. Three students finished their responses with ‘that’s all’, which, like all Conclusions, signalled the end, although no reference was made to the requirements of the task. The responses to each of the sections of the Weigh Up task were varied, as there were many appropriate ways to explain how to determine the order of boxes from heaviest to lightest. Given that this was so, it is fascinating to see that over a third of students provided a Conclusion in their responses to the Explanation section. It may well be that the imaginary nature of the last part of this task – ‘explain it to some one else in your class’ – and the fact that students were giving their third version of this explanation encouraged more students to complete their responses with Conclusions.

When considering the results in Table 8.6, there appear to be few differences according to gender. In the Better Buy task, slightly more boys used Conclusions than girls, but in the Explanation section of the Weigh Up task, slightly more girls used Conclusions than boys. It would seem that Year 8 students and students at high decile schools are much more likely to use Conclusions than other groups. The large number of students who included Conclusions in their responses to the Better Buy task have had a significant effect on these results.

It was also in responses to the Better Buy task that the two types of Conclusions were seen most clearly. These were implicit and explicit Conclusions. In responses to the Better Buy task, explicit Conclusions were ones which made reference to better value for money, which was what the original question had asked about. An implicit Conclusion did not make direct reference to better value, but instead made a veiled reference through using words such as 'only' or 'cheaper'. In the responses to the Better Buy task, boys and Year 8 students and students from high decile schools were more likely to use explicit Conclusions, whilst implicit Conclusions were used by equivalent numbers of boys and girls, but also by more Year 8 students and students attending high decile schools. More students used implicit Conclusions than explicit ones in this task.

In the remaining task, it was not always easy to distinguish between these two types of Conclusions. However, on the whole, more implicit Conclusions were used than explicit Conclusions. This was the case in the responses to all three parts of the Weigh Up task as can be seen in Tables 4.2, 4.15 and 4.28. In these responses, students were far more likely to mention the boxes being 'in order', which was coded as implicit, than to say something about the boxes being 'in order from lightest to heaviest', which would have been coded as an explicit Conclusion.

Logical connectives were also important in joining Consequences to Conclusions in responses to only some of the tasks. Given that extra information for the backing for the claims in the Conclusions could be provided in these logical connectives, it is surprising to not find them in this position in responses to all tasks. In responses to the Better Buy task, though 'and' was still the most often used connective, 6 students used 'so' and a further 5 used 'but'. These causal connectives were more likely to be used by Year 8 students (as also were the narrative connectives in this position) and least likely by students attending low decile schools. In the responses to the Plan and Description parts of the Weigh Up task, very few connectives were used between Consequences and Conclusions. In the responses to the Explanation section, 23 students used connectives, but only seven of these were causal. As had been the case with logical connectives surrounding Consequences, more girls than boys used narrative connectives. In the Motorway task, only 7 students used connectives before the Conclusion and, of these, none were causal. There were not enough Conclusions in the Bank Account task for logical connectives to be worth considering.

It would seem that Conclusions were an optional element for many students when giving mathematical explanations and justifications. However, for some tasks, like Better Buy, the majority of students felt that their responses should include a Conclusion of some sort. When Conclusions were given, they were usually implicit, with the listener left to make the final jump between the speaker's response and the actual question asked. As backings are considered to be the information which defined when the claim is true, it was felt that the logical connectives would be the linguistic embodiment of these backings, particularly around Consequences. As Conclusions represented the claims themselves, it was also felt that the logical connectives around them could also constitute part of this backing. However, on the whole very few connectives were used and mostly these were narrative rather than causal. This suggests that it was uncommon for students in primary schools to use causal connectives to identify the conditions under which claims were valid.

Suppositions

Suppositions could not be related to any of the components of explanations described by Krummheuer (1997). Instead, this text element was used to add a propositional element. Suppositions were clauses which made preceding or following clauses propositional or more vague. As a result, they never came at the beginning or the end of an explanation. Although Suppositions were only coded within the Weigh Up task, they were also evident in responses to the Bank Account and Motorway tasks. However, they were considered as hedges and are described in the sections on hesitant language, at the end of the chapters on these tasks.

In the Weigh Up task, students needed to talk about courses of action that they may take rather than those that they had taken. Using Suppositions gave students ways to discuss these possible courses of action. This enabled students to give the generic responses described by Bills and Grey (2001) which used a specific example to illustrate a general case. This is the situation in 'say if C was the heaviest you stick that, that one down at the bottom'. 'Say' was coded as a supposition, as it situates what follows as a possibility rather than a definite action.

Table 8.7. ***Use of Suppositions in different tasks.***

Suppositions	Gender		Year Level		School Decile			Total
	Girls	Boys	Year 4	Year 8	Low	Middle	High	
WU Plan	10	5	3	12	6	6	3	15
WU Description	9	5	6	8	1	8	5	14
WU Explanation	7	6	4	9	2	6	5	13

Table 8.7 shows that there were very few distinctions across tasks. This is surprising, as the Description part of this task required the students to describe what they were doing rather than what they would be doing. It was not expected that there would be uncertainty in this. However, in responses to this part of the task, the Suppositions used were predominantly 'I think' which lessens the certainty of what was proposed, as was the case in 'I think that one there's the second heaviest'. These did not set up a proposition in the same way as 'say' did.

There were some differences in the groups who used Suppositions as girls and Year 8 students used them more than other groups. It would also seem that students attending middle decile schools were slightly more likely to use them than students attending other decile levels of schools.

Suppositions added uncertainty to information contained in other text elements. Sometimes they were used by students to lessen a possible loss of face, whilst at other times they were used to show that an action was proposed rather than actually happening.

Elaborators

Elaborators like Suppositions were not used in responses to all tasks. They added further information to other text elements and could be considered as contributing to backings given in mathematical arguments as sometimes the extra information was about the conditions for when the claim was valid. This can be seen in the following example: 'test for the lightest one and then put the two left overs in the middle whichever's the heaviest obviously goes that way'. 'Whichever's the heaviest' is the Elaborator and provides information which constrains what 'goes that way'.

Elaborators were combined with all other text elements but they were most likely to be combined with Premises and Consequences, which were the two most commonly used text elements. Table 8.8 provides information on who used Elaborators in which task.

Table 8.8. ***Use of Elaborators in different tasks.***

Elaborators	Gender		Year Level		School Decile			Total
	Girls	Boys	Year 4	Year 8	Low		High	
WU Plan	25	21	18	28	28		14	46
WU Description	18	20	17	21	21		11	38
WU Explanation	34	30	30	34	34		18	64
Bank Account	15	8	10	13	13		7	23

Unlike Suppositions, there do seem to be task differences in the number of students who used Elaborators. 64 out of a possible 71 students used Elaborators in the Explanation section of the Weigh Up task. As this section produced the longest, most complex responses, it would seem that Elaborators contributed to this complexity. However, the responses to the Bank Account task were also long but often the text structures were quite simple compared to those found in the responses to this section of the Weigh Up task. Only 23 students used Elaborators in the Bank Account task and these were mostly used with Premises, as can be seen in the following two tables.

Tables 8.9 and 8.10 show how Elaborators were distributed with Premises and Consequences. Although some Introductions, Physical Consequences and Conclusions included Elaborators, there were so few of these that they have not been included.

Table 8.9. ***Use of Elaborators with Premises.***

	Gender		Year Level		School Decile			Total
	Girls	Boys	Year 4	Year 8	Low		High	
WU Plan	12/35	6/31	7/31	11/35	8/22		5/23	18/66
WU Description	9/27	5/27	6/25	7/29	5/16		4/21	14/54
WU Explanation	24/36	21/34	18/34	27/36	16/23		13/24	45/70
Bank Account	12/36	6/36	8/36	10/36	6/24		7/24	18/72

Table 8.9 shows that only in responses to the Explanation section of the Weigh Up task were Elaborators combined with Premises by the majority of students. In the other tasks, about a quarter of students combined Premises with Elaborators. When all the results are considered together, there seems to be little difference between Year 4 and 8 students or students attending different decile levels of schools. Although Premises were used equally by boys and girls, girls were more likely to combine them with Elaborators.

Table 8.10. ***Use of Elaborators with Consequences.***

	Gender		Year Level		School Decile			Total
	Girls	Boys	Year 4	Year 8	Low		High	
WU Plan	18/20	17/26	19/24	16/22	10/13		14/17	35/46
WU Description	12/25	14/25	13/22	13/28	9/15		13/19	26/50
WU Explanation	29/34	25/30	25/30	29/34	18/18		20/24	54/64
Bank Account	6/17	3/17	5/19	4/15	2/11		6/13	9/34

As was noted in the Weigh Up chapter, a greater proportion of Consequences were combined with Elaborators than Premises. In the responses to the Bank Account task, this was not the case. Slightly more girls than boys used Elaborators with Consequences but the distinction is quite small. There does not seem to be any distinction between age level or decile level of school attended. The results of Tables 4.14, 4.27 and 4.41 suggest that there is no clear relationship between using an elaborator with a Consequence and having a response considered clear and accurate.

Elaborators were text elements which only appeared in students' responses when combined with another text element. Their role was to provide further information than was already given in the text element with which they were combined. Proportionally, they were used with Consequences more often than any other text elements. When Elaborators were combined with Consequences, they could also provide information about when the warrants given in the Consequences were valid and thus could be considered as part of the backings.

Combining text elements

The distribution of text elements throughout students' responses to the four mathematics assessment tasks was not random. Although there were always some exceptions, on the whole, Introductions began the responses, Premises preceded Consequences which preceded Conclusions. Suppositions and Elaborators were always combined with other elements. Premises, Consequences and Premise – Consequence combinations (with their corresponding Elaborators and Suppositions) could be iterated several times throughout a response. Figure 8.1 provides a flowchart of the possible decision making process that a student goes through subconsciously in order to provide a response to mathematical assessment tasks.

This flowchart is idealised in two ways. The first is that it is unlikely that even unconsciously students would go through such a decision making process. The other way in which it is idealised is that the sort of response that such a flowchart would produce would match very few of the actual examples that were given. This is because there was so much variation in the responses. Some of this variation could be

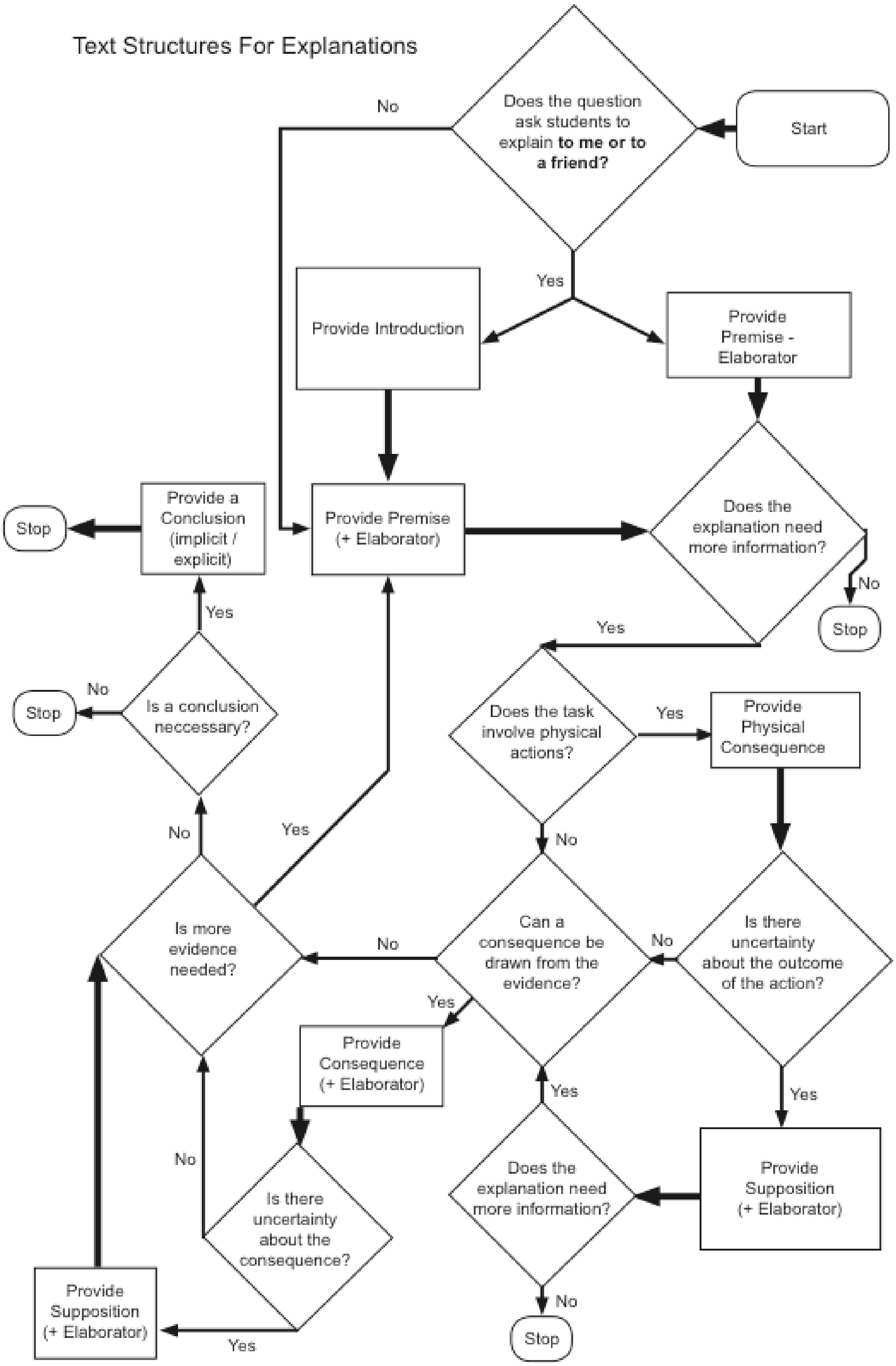


Figure 8.1: Flowchart showing steps to forming a mathematical explanation.

accounted for in regard to the accuracy of the responses, as certain combinations such as Premise – Consequence or Physical Consequence – Consequence combinations were more likely to be used in accurate responses. However, there were differences in the likelihood of these combinations appearing in accurate responses, depending on which task students were responding to. This suggests that there was more to knowing how to provide appropriate mathematical explanations and justifications than knowing how to combine text elements. This is discussed further in the next section.

Contextual Configuration

It was suggested in Figure 1.1, that in order to be assessed as accurate in responding to mathematical tasks, students need to have knowledge of mathematics and mathematical language. This research suggests that what is required is far more complex. Although most groups of students do seem to have appropriate text elements in their linguistic repertoire, these were not always used in a mathematically appropriate way. Bills (2002) had also found that, although students had certain linguistic features when discussing non-mathematical topics, they did not always chose to use them when discussing mathematical topics. Bills (2002) suggested that, when students were accurate, they were more likely to use certain features such as logical connectives. However, our research suggests that there were some students who provided accurate answers in tasks such as Motorway, but were unable to provide accurate descriptions of their thinking processes. This can be seen in the following example:

> Q
> Is it an estimation?
> Q
> ....(counting to herself)......one hundred and, oh .. umm, nine hundred and ninety.
> Q
> Because each car goes, every 98 cars every minute so then make it so it's, then you go to one hundred and eighty, and then one hundred and ninety and one hundred and eighty, you go until you get to ninety something (voice trailing off at the end).

In trying to understand the variety of students' responses, the relationship between the contextual configuration and the text structure needs further examination. In cases such as shop encounters, Hasan (Halliday & Hasan, 1985) suggested that there is a clear relationship between the contextual configuration and the text structure. The compulsory text elements such as the buyer's description of what they want comes at a set place in the interaction, along with optional elements such as the vendor's enquiry about the buyer's needs. The contextual configuration was such that the roles of vendor and buyer are clearly delineated by the event of buying something, which restricts significantly what is appropriate for each participant to say. Krummheuer's (1997) description of the components of mathematical arguments suggested that certain features would be evident in mathematical explanations and justifications. This would thus also provide a clear relationship between the contextual configuration and the text structure. It would seem that Premises, Consequences, Conclusions and Elaborators can be related to grounds, warrants, claims and backings at least in a loose way. It would also seem, that in giving responses to mathematics assessment tasks, primary school children draw upon this set of text elements. However, not all of these text elements would be specific to mathematical assessments. Nor were some text elements and combinations likely to provide appropriate mathematical responses. For example, students who used personal or factual Premises were unlikely to be giving accurate responses.

So what influenced students to give appropriate responses? There is a relationship between the contextual configuration and text structure and this can provide information on the stage that these students were at in giving appropriate mathematical responses. In order to act appropriately in these situations, students needed first to recognise the contextual configuration that they were in and then draw upon three interconnected pieces of knowledge: appropriate mathematical knowledge; knowledge of how to interact with a teacher; and knowledge of how to structure mathematical explanations. These can be seen in Figure 8.2.

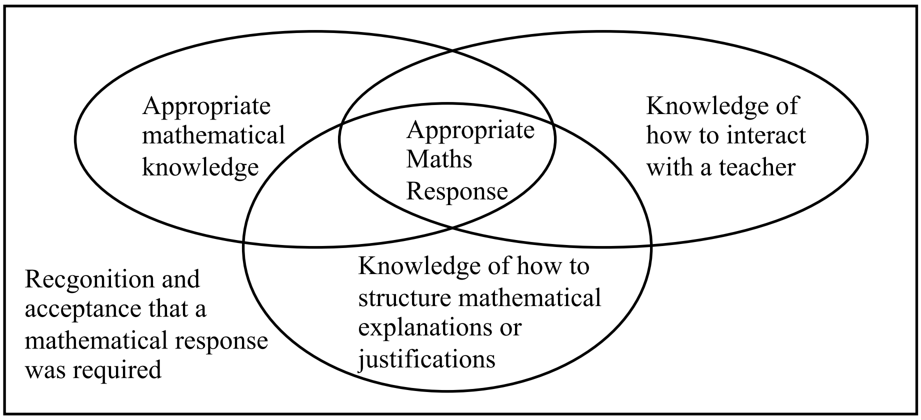


Figure 8.2: Knowledge drawn upon to produce an appropriate response.

If students did not recognise that there was a need to show their mathematical knowledge in the interaction, then it is unlikely that students drew upon their knowledge of mathematics or the structure of mathematical explanations (as discussed in the previous chapter on story telling). This can be seen in the following response from the Better Buy task by a Year 4 girl from a low decile school:

> Because you couldn't, it's easy if you had two kids you could get them to share two, and a half each, because if they had this half, one and a half each they'd complain that one was bigger and like that, one was bigger and one was smaller.

This girl used many of the features of mathematical explanations such as a Premise – Consequence combination joined with causal connectives. However, the mistaken belief that the task required a description of a shopping strategy means that the response cannot be considered appropriate, as it did not show what she knew mathematically.

If students recognised the purpose of the interaction as one requiring them to show what they knew mathematically, they still needed to combine their understanding in the other three areas to produce an appropriate response. The responses documented in this report suggest that it is not often a lack of knowledge of how to interact appropriately with a teacher which inhibited students' production of mathematical responses. Yet there were instances where students stated that they did not know the answer or refused to answer altogether, which could be seen as a breaking of the pragmatic rules of conversation. In the responses to the Bank Account task, of the 12 students who gave these sorts of responses, only one was from a high decile school. This task was difficult for all students, so it was interesting to see that these were not considered valid responses by students attending high decile schools.

Often, these sort of responses could be seen as occurring foremost because of the students' lack of mathematical knowledge. However, this was not always the case as can be seen in the following interaction between the teacher administrator and a Year 8 boy from a high decile school:

Q
.....I don’t know.
Q
Nine hundred.
Q
Um, because it says nine minutes so I just went up to nine hundred.

Although his initial response is that he did not know, when questioned further by the teacher administrator, he was quite capable of doing the mathematics. It is hard to know why he chose not to show what he was capable of initially. Maybe aspects of the situation such as receiving no feedback on the accuracy of his performance confused him about what was required from him by the teacher administrator. It may be that in the classroom in which he had learnt about acceptable responses in similar situations, that confessions of ignorance were acceptable to his teacher. They, therefore, did not break the pragmatic rule of providing a response which fitted the hearer’s expectations. Conversely, it may be that, although he knew what was required of him, he did not share the teacher’s belief that it was necessary for him to do this. In this case, although the teacher administrator may believe the contextual configuration is fixed, the student feel that other options, such as non-compliance are available.

The third area of knowledge that students need in order to provide an appropriate mathematical explanation or justification was knowledge of the appropriate text structures. The responses examined in this report suggest that acceptable responses for this age group, although drawn from a restricted set of text elements, can be extremely variable. As has been reported in each chapter, there are some text combinations, such as Premise – Consequence, which seem to be correlated with accurate and clear responses. However, in the previous example, this boy did not explain how he gained this answer clearly, even though he uses a Premise – Consequence combination joined with a causal connective. It may be that the range of proof schemes which are appropriate in primary school in responding to these different tasks encourages such diversity in text structures, even when students have sufficient mathematical knowledge. At later stages in their school careers, it may be that they are restricted to analytical proof schemes which Sowder and Harel (1998) described as ‘the ultimate in justifications’ and thus the text structures are also restricted. Further investigation needs to occur to determine the validity of this conjecture.

When students’ responses are considered holistically, it can be seen that more than just mathematical understanding and being able to structure a mathematical response is required. This includes having an awareness of what is involved in the contextual configuration and accepting it as valid. Once students have recognised and agreed on the need to show their mathematical knowledge, they then need to access and combine the appropriate mathematical knowledge, ways of interacting with a teacher administrator and knowledge of text structures of mathematical explanations and justifications.

Conclusion

In the responses to the different mathematical tasks, students combined a series of text elements (Introductions, Premises, Consequences, Conclusions, Suppositions and Elaborators) into text structures. Many of these text elements could be related to the constituent parts that Krummheuer (1997) proposed for a mathematical argument. It was also possible to see a relationship between the type of Premise provided and the proof schemes described by Sowder and Harel (1998). This suggested that the contextual configuration had an effect on the text structures needed for an appropriate mathematical explanation and justification. The flowchart given in Figure 8.1 suggested a series of decision making points that students went through in producing their responses to the different assessment tasks.

However, it was also clear that there was immense variability in the responses, so it may not just be mathematical or language ability alone which determines whether a student produced an appropriate mathematical response. It would seem that not only should the contextual configuration be recognised by both participants – the student and the teacher administrator – but also it must be accepted by the student as valid for them to then draw upon their mathematical knowledge, language knowledge and interactional knowledge.

Chapter 9
Implications for Teachers

Although the linguistic analysis in this probe study provides more information than is needed by classroom teachers, there are valuable points that would enable teachers to improve the ability of their students to give mathematical explanations.

Primary among these is to encourage their students to talk mathematics rather than just doing it. This advice is not new. It is an underpinning of the New Zealand Numeracy Project (Ministry of Education, 2004) as well as the NCTM Standards (2000) and several research projects (for example Khisty and Chval, 2002). Talking mathematics provides an opportunity for students to formulate their own understanding, explain it to their peers and explain their understanding to their teachers.

The major linguistic structures identified here, Premise, Consequence, and Conclusion, can be used in many types of mathematical discussion. This can be discussion among peers, with a teacher, and in writing. Evaluation of discussion in mathematics classes can be guided by seeing whether or not the appropriate elements are present. Teachers can guide students by asking them to explain in complete sentences, explain what they already know, what alternatives might eventuate, and what can be concluded from different outcomes. This structure can be applied to numerical investigations, algebraic comparisons, investigations involving statistics and probability, and geometric and measurement hypotheses and investigations. Not only are they useful for all strands of the curriculum, but for both primary and secondary students. Students taking the National Certificate of Educational Achievement will need to be able to write explanations including these components in order to achieve Excellence.

Primary school students need to be helped to tell the difference between a story appropriate for reading, writing or fantasy and one appropriate for mathematics. Many current pedagogical practices make this differentiation more difficult for children, for example calling equations “number sentences” and calling word problems “story problems”. This does not mean that fun and fantasy need to be removed from mathematics. Exercises in the size of a giant’s hands or cloths are both fun and mathematically useful exercises in proportion. However, the subject of a mathematical explanation for this as well as for other mathematical tasks is a clear explanation involving stating a Premise, Consequence and Conclusion.

References

Anthony, G. & Walshaw, M. (2002). *Students' conjectures and justifications: Report of a Probe Study carried out for the National Education Monitoring Project*. Palmerston North: Massey University.

Barron, N. (1971). Sex-typed language: The production of grammatical cases. *Acta sociologica 14*, 24-42.

Bernstein, B. B. (1975). Language and socialization. In S. Rogers (Ed), *Children and language Readings in early language and socialization.* London: Oxford University Press.

Bernstein, B. (1990). *The structuring of pedagogic discourse, Volume 4: Class, codes and control*. London: Routledge.

Bicknell, B. (1999). The writing of explanations and justifications in mathematics: Differences and dilemmas. In J. M. Truran & K. M. Truran (Eds), *Making the Difference: Proceedings of the 22nd Annual Conference of the Mathematics Education Research Group of Australasia Incorporated* (pp. 75-83). Adelaide: Mathematics Education Research Group of Australasia.

Bills, C. (2002). Linguistic pointers in young children's descriptions of mental calculations. In A Cockburn & E. Nardi (Eds.), *Proceedings of the 26th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 97-104). Norwich: PME.

Bills, C., & Grey, E. (2001). The 'particular', 'generic' and 'general' in young children's mental calculations. In M. van den Heuval-Panhuizen (Ed.), *Proceedings of the 25th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 153-160). Utrecht: PME.

Bogdan, R. C. & Biklen, S. K. (1982). *Qualitative research for education: An introduction to theory and methods*. Boston: Allyn and Bacon Inc.

Brown, G. (1977). *Listening to Spoken English.* London: Longman.

Brown, M. (1981). Number operations. In K. Hart, (Ed) *Children's understanding of mathematics: 11 ~ 16*. Newcastle upon Tyne: John Murray.

Brown, P. & Levinson, S. (1978) Universals in language usage: Politeness phenomena. In E. N. Goody (Ed), *Questions and Politeness: Strategies in Social Interactions* (pp. 56-289). Cambridge: Cambridge University Press.

Burns, M. (1985). The role of questioning. *The Arithmetic Teacher 32*(6), 14-17.

Burton, L. (2001). Fables: The tortoise? The hare? The mathematically underachieving male? In B. Atweh, H. Forgasz & B. Nebres (Eds.), *Sociocultural research on mathematics education: An international perspective*, (pp. 379-392). Mahwah, NJ: Lawrence Erlbaum Associates.

Burton, L. and Morgan, C. (2000). Mathematicians writing. *Journal for Research in Mathematics Education 31*(4), 429-453.

Cameron, D., Frazer, E., Harvey, P., Rampton, M. B. H. & Richardson, K. (1994). The relations between researcher and researched: Ethics, advocacy and empowerment. In D. Graddol, J. Maybin & B. Stierer (Eds), *Researching language and literacy in social contexts,* (pp. 18-25). Clevedon: Multilingual Matters.

Chapman, A. (1997). Towards a model of language shifts in mathematics learning. *Mathematics Education Research Journal 9*(2), 152-172.

Chomsky, N. (1965). *Aspects of the theory of syntax*. Cambridge, MA: MIT Press.

Christie, M. (1985). *Aboriginal perspectives on experience and learning: The role of language in Aboriginal education*. Melbourne: Deakin University Press.

Clarke, D. J. (1993). The language of assessment. In M. Stephens, A. Waywood, D. Clarke, J. Izard (Eds.), *Communicating mathematics: Perspectives from classroom practice and current research* (pp. 211-222). Melbourne: The Australian Council for Educational Research.

Coates, J. (1993). *Women, men and language*. London: Longman.

Coates, J. (2004). The acquisition of gender-differentiated language. In K. Trott, S. Dobbinson & P. Griffiths (Eds), *The child language reader*. London: Routledge.

Crooks, T. and Flockton, L. (2001). *Mathematics: Assessment Results.* Dunedin: Educational Assessment Research Unit.

Cummins, J. (1989). *Empowering minority students.* Sacramento, CA: California Association for Bilingual Education.

Deuchar, M. (1990). A pragmatic account of women's use of standard speech. In J. Coates & D. Cameron (Eds), *Women in their speech communities: New perspectives on language and sex* (pp. 27-32). New York: Longman.

Donaldson, M. (1986). *Children's explanations*. Cambridge: Cambridge University Press.

Edwards, A. D. (1976). *Language in culture and class.* London: Heinemann Educational Books.

Edwards, D. & Mercer, N. (1987). *Common knowledge: The development of understanding in the classroom*. London: Methuen.

Ellerton, N. and Clarkson, P. (1996). Language factors in mathematics teaching and learning. In A. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick and C. Laborde (Eds.), *International Handbook of Mathematics Education*, (pp. 987-1023). Dordrecht: Kluwer Academic Publishers.

Esty, W. (1992). Language concepts of mathematics. *Focus on learning problems in mathematics 14*(4), 31-54.

Fishman, P. M. (1978). Interaction: The work women do. *Social Problems 25*, 397-406.

Flockton, L. & Crooks, T. (1997). *Mathematics: Assessment Results.* Dunedin: Educational Assessment Research Unit.

Forman, E., Larreamendy-Joerns, J., Stein, M. K. & Browns, C. A. (1998). 'You're going to want to find out which and prove it': Collective argumentation in a mathematics classroom. *Learning and Instruction 8*(6), 527-548.

Gooding, A. & Stacey, K. (1993). Characteristics of small group discussion reducing misconceptions. *Mathematics Education Research Journal 5*(1), 60-73.

Grob, L. M., Meyers, R. A. & Schuh, R. (1997). Powerful/powerless language use in group interactions: Sex differences or similarities? *Communication Quarterly 45*(3), 282-304.

Haig, Y. and Oliver, R. (2003). Language variation and education: Teachers' perceptions. *Language and Education 17*(4), 266-280.

Harris, S. (1980). *Culture and learning: Tradition and education in Northeast Arnhemland*, Darwin: Northern Territory Department of Education.

Hawkins, P. R. (1977). *Social class, the nominal group and verbal strategies*. London: Routledge & Kegan Paul.

Heath, S. B. (1982). Questioning at home and at school: A comparative study. In G. Spindler (Ed), *Doing the ethnography of schooling* (pp. 102-131). New York: Holt, Rinehart & Winston.

Halliday, M. A. K. (1985). *Spoken and Written Language*. Melbourne: Deakin University Press.

Halliday, M. & Hasan, R. (1985). *Language, Context and Text: Aspects of Language in a Social-semiotic Perspective.*: Deakin University Press,.

Hass, W. A & Wepman, J. M. (1974). Dimensions of individual difference in the spoken syntax of school children. *Journal of speech and hearing research 17*, 455- 469.

Holmes, J. (1987). Hedging, fencing and other conversational gambits: An analysis of gender differences in New Zealand speech. In A. Pauwels (ed) *Women and language in Australia and New Zealand Society* (pp. 59-79). Sydney: Australian Professional Publishers.

Hudson, J. A. & Shapiro, L. R. (1991). From knowing to telling: The development of children's scripts, stories and personal narratives. In A. McCabe & C. Peterson (Eds). *Developing narrative structure* (pp. 89-136) Hillside, NJ: Lawrence Erlbaum Associates .

Hymes, D. (1972). On communicative competence. In J. B. Pride & J. Holmes (Eds), *Sociolinguistics* (pp. 269-293). Harmondsworth: Penguin.

Irwin, K. C. & Niederer, K (2002). *An evaluation of the Numeracy Exploratory Study, Years 7 – 10, 2001*, Wellington: Ministry of Education.

Irwin, K. C & Ginsburg, H. P. (2001). Early mathematical discourse. In M van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education.* Utrecht: PME.

Jones, A. (1988). Which girls are 'Learning to Lose'? In S. Middleton (Ed), *Women and education in Aotearoa* (pp. 143-160). Sydney: Allen & Unwin.

Kaput, J. J. (1988). Linking representations in the symbol systems of algebra. In S. Wagner & C. Kieran (Eds), *Research issues in learning and teaching algebra: Vol 4* (pp. 167-194). Reston, VA: NCTM.

Khisty, L. L. & Chval, K. B. (2002). Pedagogic discourse and equity in mathematics: When teachers' talk matters. *Mathematics Education Research Journal*, *14*(3), 154-168.

Koehler, M. S. (1990). Classrooms, teachers and gender differences in mathematics. In E. Fennema & G. C. Leder (Eds), *Mathematics and gender* (pp. 128-148). Brisbane: University of Queensland Press.

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 229-270). Hillsdale, NJ: Lawrence Erlbaum.

Labov, W. (1966). *The social stratification of English in New York City*. Washington, DC: Centre for Applied Linguistics.

Labov, W. (1990). The intersection of sex and social class in the course of linguistic change. *Language variation and change 2*, 205-254.

Langvogt, J., Leder, G. and Forgasz, H. (2002) Sugar, spice and puppy dog tails: Gendered perceptions of talent and high achievement. In W. Vialle, & J. Geake

(Eds), *The gifted enigma: a collection of articles*, (pp. 403-431). Melbourne: Hawker Brownlow Education.

Laver, J. & Trugill, P. (1979). Phonetic and linguistic markers in speech. In K. R. Scherer & H. Giles (Eds), *Social markers in speech* (pp. 1-32) Cambridge: Cambridge University Press.

Learning Media (1992). *Mathematics in the New Zealand Curriculum* Wellington: Learning Media.

Martino, A. M. & Maher, C. A. (1994). *Teacher questioning to stimulate justification and generalization in mathematics*. Paper presented at the Annual Meeting of the American Educational Research Association (New Orleans, April 4-8, 1994).

McCabe, A. & Peterson, C. (1985). A naturalistic study of the production of causal connectives by children. *Journal of child language 12*, 145-159.

Malcolm, I. (1982). Speech events of the Aboriginal classroom. *International Journal of Sociology of Language*, *36*, 115-134.

Meaney, T. (2002). Aspects of written performance in mathematics learning. In K. Irwin, B. Barton, M. Pfannkuch, M. Thomas (Eds), *Mathematics Education in the South Pacific* (Proceeding of the 25th Annual Conference of the Mathematics Education Research Group of Australasia). Auckland: MERGA.

Meaney, T. (2005, forthcoming). Mathematics as Text. In A. Chronaki & I. M. Christiansen (Eds), *Challenging Ways of Viewing Maths Classroom Communication*. Westport, CT: Information Age.

Meyerhoff, M. (1987). Language and sex: Research in New Zealand. In A. Pauwels (Ed), *Women and language in Australia and New Zealand Society* (pp. 32-44) Sydney: Australian Professional Publishers.

Milroy, L. (1987). *Observing and analysing natural language: A critical account of sociolinguistic method*. Oxford: Basil Blackwell.

Ministry of Education (2004). *New Zealand Maths Numeracy Projects.* Retrieved 21 Jan. 2005 http://www.nzmaths.co.nz/numeracy/2005numPDFs/pdfs.htm

Morgan, C. (1998). *Writing Mathematically: The Discourse of Investigation*. London: Falmer Press.

Moskal, B. M. & Magone, M. E. (2000) Making sense of what students know: Examining the referents, relationships and modes students displayed in response to a decimal task. *Educational Studies in Mathematics 43*, 311-335.

National Council of Teachers of Mathematics (2000). Principles and standards for school mathematics. Reston VA: National Council of Teachers of Mathematics.

Peterson, C. & McCabe, A. (1991). Linking children's connective use and narrative macrostructure. In A. McCabe & C. Peterson (Eds), *Developing narrative structure* (pp. 29-54). Hillside, NJ: Lawrence Erlbaum Associates.

Piaget, J. (1928). *Judgement and reasoning in the child*. London: Routledge & Kegan Paul.

Pimm, D. & Wagner, D. (2003). Investigation, mathematics education and genre: An essay review of Candia Morgan's "Writing mathematically: The discourse of investigation". *Educational studies in mathematics*, *55*, 159-178.

Robinson, W. P. (1979). Speech markers and social class. In K. R. Scherer & H. Giles (Eds), *Social markers in speech* (pp. 211-249). Cambridge: Cambridge University Press.

Rowland, T. (1995). *Vagueness in mathematics talk*. Unpublished PhD thesis, Open University.

Rowland, T. (2000). *The pragmatics of mathematics education: Vagueness in mathematical discourse* London: Falmer Press.

Sacks, H. (1992). *Lectures on conversation, Vol 1* G. Jefferson & E. A. Schegloff, (Eds.) Oxford: Blackwell.

Sierpinska, A. (1999). The notion of the didactic contract. Lecture notes on the theory of didactic situations. Retrieved 21 July 2004 http://alcor.concordia.ca/~sierp/TDSLecture%203.pdf

Silver, E. A., Shapiro, L. J., & Deutsch, A. (1993). Sense making and the solution of division problems involving remainders: An examination of middle school students' solution processes and their interpretations of solutions*, Journal for Research in Mathematics Education,24(2),* 117-135.

Sowder, L. & Harel, G. (1998). Types of students' justifications. *The mathematics teacher 91*(8), 670-675.

Swann, J. (1994). Observing and recording talk in educational settings. In D. Graddol, J. Maybin & B. Stierer (Eds), *Researching language and literacy in social contexts* (pp. 26-48). Clevedon: Multilingual Matters.

Swann, J., & Gaddol, D. (1994). Gender inequalities in classroom talk. In D. Gaddol & B. Stierer (Eds.), *Researching language and literacy in social contexts*. (pp. 151-167). Clevedon PA: Multilingual Matters.

Thomas, S. N. (1973). *Practical reasoning in natural language*. Englewood Cliffs, NJ: Prentice-Hall.

Toulmin, S. (1969). *The uses of argument*. Cambridge: Cambridge University Press.

Unsworth, L. (2001). *Teaching multiliteracies across the curriculum*. Buckingham: Open University Press.

Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems*. Lisse: Swets & Zeitlinger.

Wickett, M. (1997). Uncovering bias in the classroom – a personal journey. In J. Trentacosta & M. J. Kenney (Eds), *Multicultural and gender equity in the mathematics classroom: 1997 Yearbook* Reston, VA: The National Council of Teachers of Mathematics.

Wodak, R. & Benke, G. (1997). Gender as a sociolinguistic variable: New perspectives on variation studies. In F. Coulmas (Ed), *The handbook of sociolinguistics* (pp. 127-150). Oxford: Blackwell Publishers.

Yackel, E. (2001) Explanation, justification and argumentation in mathematics classrooms. In A Cockburn & E. Nardi (Eds), *Proceedings of the 26th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 9- 25). Norwich: PME.

Young-Loveridge, J. (2000). How children's understanding of the number system varies as a function of ethnicity and socio-economic status. In J. M. Bana, J. & A. Chapman (Eds), *Mathematics Education beyond 2000: Proceedings of the Twenty-Third Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 672-679). Fremantle: MERGA.

Zevenbergen, R. (2001). Mathematics, social class and linguistic capital: An analysis of mathematics classroom interactions. In B. Atweh, H. Forgasz & B. Nebres (Eds), *Sociocultural research on mathematics education: An international perspective*, (pp. 201-216). Mahwah, NJ: Lawrence Erlbaum Associates.